1.1 CONCEPT QUESTIONS

1. Explain what is meant by the statement $\lim_{x\to 2} f(x) = 3$.
2. a. If $\lim_{x\to 3} f(x) = 5$, what can you say about $f(3)$? Explain.
   b. If $f(2) = 6$, what can you say about $\lim_{x\to 2} f(x)$? Explain.
3. Explain what is meant by the statement $\lim_{x\to 3^-} f(x) = 2$.
4. Suppose $\lim_{x\to 1^-} f(x) = 3$ and $\lim_{x\to 1^+} f(x) = 4$.
   a. What can you say about $\lim_{x\to 1} f(x)$? Explain.
   b. What can you say about $f(1)$? Explain.

1. 当 x 夠靠近 2 時，f(x) 也会夠靠近 3

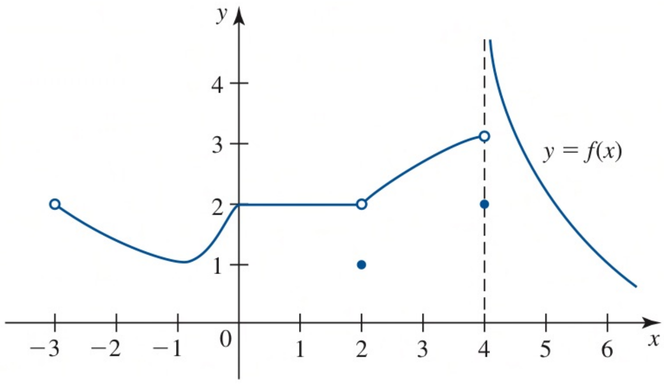
2. (a), (b) 知道該点極限值不代表可以知道該点的函数值，反之亦然

3. 当 x 從 3 的左側夠靠近 3 時，f(x) 也会夠靠近 2

4. (a) ∵ 左極限與右極限不同
∴ $\lim_{x\to 1} f(x)$ 不存在

(b) 與第二題答案相同

7. Use the graph of the function f to determine whether each statement is true or false. Explain.

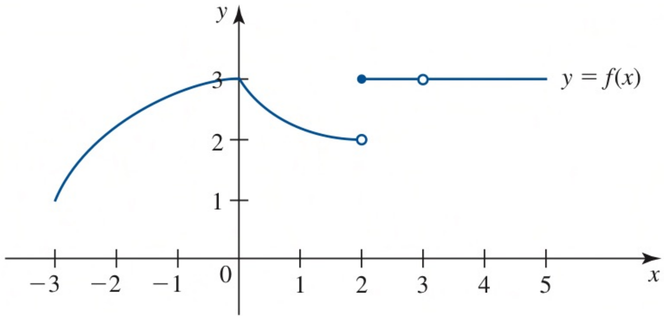


a. $\lim_{x\to -3^+} f(x) = 2$ b. $\lim_{x\to 0} f(x) = 2$

c. $\lim_{x\to 2} f(x) = 1$ d. $\lim_{x\to 4^-} f(x) = 3$

e. $\lim_{x\to 4^+} f(x)$ does not exist f. $\lim_{x\to 4} f(x) = 2$

8. Use the graph of the function f to determine whether each statement is true or false. Explain.



a. $\lim_{x\to -3^+} f(x) = 1$ b. $\lim_{x\to 0} f(x) = f(0)$

c. $\lim_{x\to 2^-} f(x) = 2$ d. $\lim_{x\to 2^+} f(x) = 3$

e. $\lim_{x\to 3} f(x)$ does not exist f. $\lim_{x\to 5^-} f(x) = 3$

7. (a) True

(b) True

∵ $\lim_{x\to0^-} f(x) = 2 = \lim_{x\to0^+} f(x)$

(c) False

∵ $\lim_{x\to2^-} f(x) = \lim_{x\to2^+} f(x) = 2$

∴ $\lim_{x\to2} f(x) = 2 \neq 1$

(d) True

(e) True ∵ $\lim_{x\to4^+} f(x) = \infty$

(f) False

∵ $\lim_{x\to4} f(x)$ doesn't exist

8. (a) True

(b) True ∵ $\lim_{x\to0^-} f(x) = \lim_{x\to0^+} f(x) = 3$

(c) True

(d) True

(e) False ∵ $\lim_{x\to3^-} f(x) = \lim_{x\to3^+} f(x) = 3 \Rightarrow \lim_{x\to3} f(x) = 3$ (exist)

(f) True

In Exercises 17–22, sketch the graph of the function f and evaluate (a) $\lim_{x\to a^-} f(x)$, (b) $\lim_{x\to a^+} f(x)$, *and* (c) $\lim_{x\to a} f(x)$ *for the given value of a.*

17. $f(x) = \begin{cases} x - 1 & \text{if } x \le 3 \\ -2x + 8 & \text{if } x > 3 \end{cases}; \quad a = 3$

18. $f(x) = \begin{cases} 2x - 4 & \text{if } x < 4 \\ x - 2 & \text{if } x \ge 4 \end{cases}; \quad a = 4$

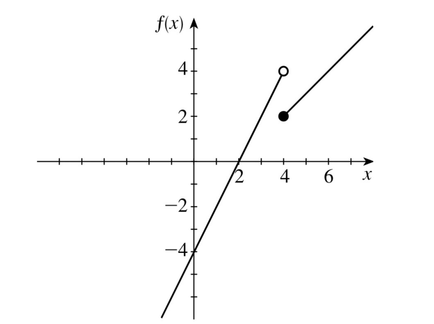
19. $f(x) = \begin{cases} -x^2 + 4 & \text{if } x \ne 0 \\ 2 & \text{if } x = 0 \end{cases}; \quad a = 0$

20. $f(x) = \begin{cases} x^2 - 1 & \text{if } x \ne 0 \\ 1 & \text{if } x = 0 \end{cases}; \quad a = 0$

21. $f(x) = \begin{cases} x & \text{if } x < 1 \\ 2 & \text{if } x = 1; \\ -x + 2 & \text{if } x > 1 \end{cases} \quad a = 1$

22. $f(x) = \begin{cases} -2x + 4 & \text{if } x < 1 \\ 4 & \text{if } x = 1; \\ x^2 + 1 & \text{if } x > 1 \end{cases} \quad a = 1$

18.



(a) $\lim_{x\to 4^-} f(x) = 2\times 4 - 4 = 4$

(b) $\lim_{x\to 4^+} f(x) = 4 - 2 = 2$

(c) By (a)(b)

$\lim_{x\to 4} f(x)$ doesn't exist

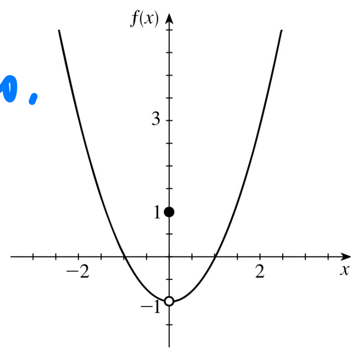
20.

(a) $\lim_{x\to 0^-} f(x) = 0^2 - 1 = -1$

(b) $\lim_{x\to 0^+} f(x) = 0^2 - 1 = -1$

(c) By (a), (b), $\lim_{x\to 0} f(x) = -1$

20.



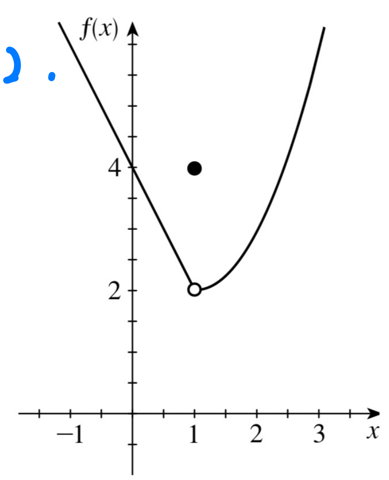
22.

(a) $\lim_{x\to 1^-} f(x) = -2\times 1 + 4 = 2$

(b) $\lim_{x\to 1^+} f(x) = 1^2 + 1 = 2$

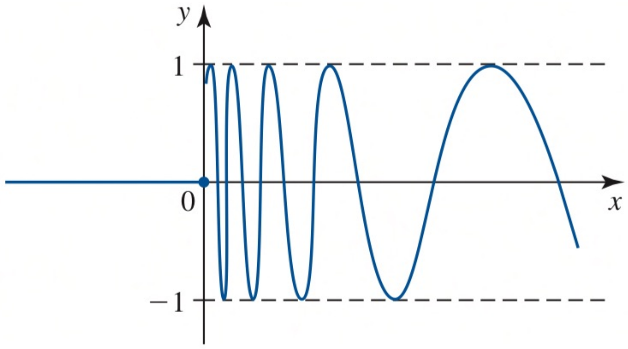
(c) By (a) (b), $\lim_{x\to 1} f(x) = 2$

22.



29. Let

$$f(x) = \begin{cases} 0 & \text{if } x \le 0 \\ \sin\dfrac{1}{x} & \text{if } x > 0 \end{cases}$$



(As x approaches 0 from the right, y oscillates more and more.) Use the figure and construct a table of values to guess at $\lim_{x\to 0^+} f(x)$, $\lim_{x\to 0^-} f(x)$, and $\lim_{x\to 0} f(x)$. Justify your answer.

x	-0.01	-0.001	-0.0001	0	0.0001	0.001	0.01
$f(x)$	0	0	0	0	-0.3056	0.8269	-0.5064

$\because \lim_{x\to 0^-} f(x) = 0$ & $\boxed{\lim_{x\to 0^+} f(x) \text{ doesn't exist}}$

一直
震盪
⇓
沒有固定值

$\therefore \lim_{x\to 0} f(x)$ doesn't exist #

31. Let

$$f(x) = \begin{cases} \dfrac{1}{x} & \text{if } x < 0 \\ \sin x & \text{if } 0 \le x < \pi \\ 0 & \text{if } x \ge \pi \end{cases}$$

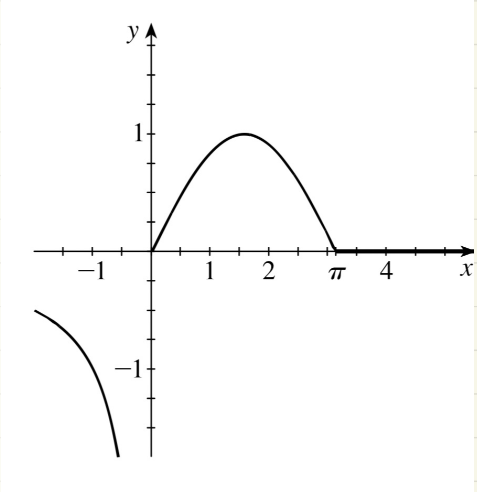
a. Sketch the graph of f.

b. Find all values of x in the domain of f at which the limit of f exists.

c. Find all values of x in the domain of f at which the left-hand limit of f exists.

d. Find all values of x in the domain of f at which the right-hand limit of f exists.

(a)



(b) $x \in (-\infty, 0) \cup (0, \infty)$

(c) $x \in (-\infty, 0) \cup (0, \infty)$

(d) $x \in (-\infty, \infty)$

In Exercises 43–46, determine whether the statement is true or false. If it is true, explain why. If it is false, explain why or give an example that shows it is false.

43. If $\lim_{x \to a} f(x) = c$, then $f(a) = c$.

44. If f is defined at a, then $\lim_{x \to a} f(x)$ exists.

45. If $\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$, then $f(a) = g(a)$.

46. If both $\lim_{x \to a^+} f(x)$ and $\lim_{x \to a^-} f(x)$ exist, then $\lim_{x \to a} f(x)$ exists.

43. False, Let $f(x) = \begin{cases} x, & \text{if } x \neq 0 \\ -1, & \text{if } x = 0 \end{cases}$.

$\Rightarrow \lim_{x \to 0} f(x) = 0$ but $f(0) = -1$

44. False. Let $f(x) = \begin{cases} \frac{1}{x}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$

$\Rightarrow f(0) = 1$ but $\lim_{x \to 0} f(x)$ doesn't exist

45. False. Let $f(x) = \begin{cases} x, & \text{if } x \neq 0 \\ -1, & \text{if } x = 0 \end{cases}$, $g(x) = x$

$\Rightarrow \lim_{x \to 0} f(x) = 0 = \lim_{x \to 0} g(x)$ but $f(0) = -1 \neq 0 = g(0)$

46. False.

Let $f(x) = \begin{cases} x, & \text{if } x \geq 1 \\ -x & \text{if } x < 1 \end{cases}$

$\Rightarrow \lim_{x \to 1^+} f(x) = 1$ & $\lim_{x \to 1^-} f(x) = -1$

both exist but not equal

Hence $\lim_{x \to 1} f(x)$ doesn't exist #