Calculus

2013/05/14

考試注意事項：

1. 答案紙直行對折，兩直欄書寫作答。
2. 無清楚計算過程，不予計分。

試題：

1. (15%) Let f defined by

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

   Show that $f_x(0,0)$ and $f_y(0,0)$ both exist but f is not differentiable at $(0,0)$.

2. (15%) Let $w = x^2y + y^2z^3$, where $x = r\cos s$, $y = r\sin s$ and $z = re^s$. Use the method of the **chain rule** to find the value of $\partial w/\partial s$ when $r = 1$ and $s = 0$.

3. (15%) Find the relative extrema of $f(x,y) = x^3 + y^2 - 2xy + 7x - 8y + 2$.

4. (15%) Use the **Lagrange multiplies** to find the extrema of the function $f(x,y) = x^2 - y^2$ subject to the given constraint $g(x,y) = x^2 + y^2 - 1 = 0$.

5. (10%) Find an equation of the plane containing the points $P(3,-1,1)$, $Q(1,4,2)$ and $R(0,1,4)$.

6. (10%) Find the length of the arc of the helix C given by the vector function $\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j} + t\mathbf{k}$, where $0 \leq t \leq 2\pi$.

7. (10%) Find equations for the tangent plane and the normal line to the surface $z = e^x \sin \pi y$ at the point $P(0,1,0)$.

8. (10%) Let $f(x,y) = x^2 - 2xy$.
   (a) Find the gradient of f at the point $(1,-2)$.
   (b) Use the result of (a) to find the directional derivative of f at $(1,-2)$ in the direction from $P(-1,2)$ to $Q(2,3)$