

考試注意事項：

1. 答案紙直行對折，兩直欄書寫作答。
2. 無清楚計算過程，不予計分。

試題：

1. (15%) Evaluate  $\int_0^1 \int_y^1 \frac{\sin x}{x} dx dy$ .

2. (15%) Evaluate

$$\iint_R \sin\left(\frac{x-y}{x+y}\right) dA$$

where  $R$  is the trapezoidal region with vertices  $(1, 0)$ ,  $(2, 0)$ ,  $(0, 2)$ , and  $(0, 1)$ .

3. (15%) Let  $\mathbf{F}(x, y) = 2xy\mathbf{i} + (1 + x^2 - y^2)\mathbf{j}$ .

(a) Show that  $\mathbf{F}$  is conservative, and find a potential function  $f$  such that  $\mathbf{F} = \nabla f$ .

(b) If  $\mathbf{F}$  is a force field, find the work done by  $\mathbf{F}$  in moving a particle along any path from  $(1, 0)$  to  $(2, 3)$ .

4. (15%) Evaluate  $\oint_C (y^2 + \tan x)dx + (x^3 + 2xy + \sqrt{y}) dy$ , where  $C$  is the circle  $x^2 + y^2 = 4$  and is oriented in the counterclockwise direction.

5. (10%) Evaluate  $\iint_R 2xy^2 dA$ , where  $R$  is the region bounded by the graphs of  $x = y^2$  and  $x = 3 - 2y^2$ .

6. (10%) Evaluate  $\iiint_T x dV$ , where  $T$  is the region bounded by  $x \geq 0$ ,  $y \geq 0$ ,  $z \geq 0$ , and lying inside the sphere  $x^2 + y^2 + z^2 = 1$ .

7. (10%) Let  $\mathbf{F}(x, y, z) = xyz\mathbf{i} + x^2y^2z\mathbf{j} + xy^2\mathbf{k}$ . Find the divergence and the curl of  $\mathbf{F}$  at the point  $(1, -1, 2)$ .

8. (10%) Evaluate  $\int_C 2x ds$ , where  $C$  consists of the arc  $C_1$  of the parabola  $y = x^2$  from  $(0, 0)$  to  $(1, 1)$  followed by the line segment  $C_2$  from  $(1, 1)$  to  $(0, 0)$ .