

考試注意事項：

1. 答案紙直行對折，兩直欄書寫作答。
2. 無清楚計算過程，不予計分。

試題：

1. (15%) Find $\lim_{n \rightarrow \infty} \frac{n!}{n^n}$, where $n! = n(n-1)(n-2)\cdots 1$.
2. (15%) Find the radius of convergence and the interval of convergence of $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^2 \cdot 3^n}$.
3. (15%) Find $\int e^{-x^2} dx$ in terms of a power series representation and determine its radius of convergence.
4. (15%) Let $f(x) = \ln x$
 - (a) Find the fourth-degree Taylor polynomial of f at $c = 1$, and use it to approximate $\ln 1.1$.
 - (b) Estimate the accuracy of the approximation that you obtain in part (a).
5. (5% \times 8) Determine whether the series is convergent or divergent.

$$(a) \sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$$

$$(b) \sum_{n=1}^{\infty} \frac{2n^2 + n}{\sqrt{4n^7 + 3}}$$

$$(c) \sum_{n=1}^{\infty} \frac{\sin 2n}{n^2}$$

$$(d) \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2^{n+3}}{(n+1)^n}$$

$$(e) \sum_{n=1}^{\infty} \frac{2n-1}{3n+1}$$

$$(f) \sum_{n=1}^{\infty} \frac{1}{n\sqrt{\ln n}}$$

$$(g) \sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2 + 1}$$

$$(h) \sum_{n=1}^{\infty} \frac{2^n}{n! n}$$