

考試注意事項：

1. 答案紙直行對折，兩直欄書寫作答。
2. 無清楚計算過程，不予計分。

試題：

1. (15%) Find the radius of convergence and the interval of convergence of $\sum_{n=1}^{\infty} \frac{x^n}{n}$.
2. (15%) Find a power series representation for $\frac{1}{(1+x)^2}$ on $(-1, 1)$ by differentiating a power series of $f(x) = -\frac{1}{1+x}$.
3. (15%) Find the Maclaurin series of $f(x) = \sin x$, and determine its interval of convergence.
4. (15%) Let $f(x) = \ln x$
 - (a) Find the fourth-degree Taylor polynomial $P_4(x)$ of f at $c = 1$.
 - (b) Let $R_4(x) = f(x) - P_4(x)$. Estimate the accuracy $R_4(1.1)$ of the approximation that you obtain in part (a).
5. (10% \times 4) Determine whether the series converges or diverges.

(a) $\sum_{n=1}^{\infty} \frac{2n^2 + 1}{3n^2 - 1}$

(b) $\sum_{n=1}^{\infty} n^{-1.001}$

(c) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + 1}$

(d) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{3n}{4n^2 - 1}$