2.7

4. $x^2y+2xy^2-x+3=0 \Rightarrow 2xy+x^2y'+2y^2+4xyy'-1=0$

$$\Rightarrow \left(x^2+4xy\right)y' = 1-2xy-2y^2 \Rightarrow y' = \frac{1-2xy-2y^2}{x\,(x+4y)}$$

12. $\left(2x^2+3y^2\right)^{5/2} = x \Rightarrow \left(2x^2+3y^2\right) = x^{2/5} \Rightarrow 4x+6yy' = \frac{2}{5}x^{-3/5}$

$$\Rightarrow 6yy' = \frac{2}{5x^{3/5}} - 4x = \frac{2\left(1-10x^{8/5}\right)}{5x^{3/5}} \Rightarrow y' = \frac{1-10x^{8/5}}{15x^{3/5}y}$$

16. $x+y^2 = \cos xy \Rightarrow 1+2yy' = (-\sin xy)\left(y+xy'\right)$

$$\Rightarrow (2y+x\sin xy)\,y' = -y\sin xy - 1 \Rightarrow y' = -\frac{y\sin xy+1}{2y+x\sin xy}$$

22. $x^2y+y^3=2 \Rightarrow 2xy+x^2y'+3y^2y'=0 \Rightarrow y' = -\frac{2xy}{x^2+3y^2}$, so $y'\big|_{(-1,1)} = -\frac{2\,(-1)\,(1)}{1+3} = \frac{1}{2}$.

An equation of the tangent line is $y-1=\frac{1}{2}(x+1)$ or $y=\frac{1}{2}x+\frac{3}{2}$.

24. $y=\sin xy \Rightarrow y' = (\cos xy)\left(y+xy'\right) \Rightarrow y' = \frac{y\cos xy}{1-x\cos xy} \Rightarrow y'\big|_{(\pi/2,1)} = 0$.

An equation of the tangent line is $y-1=0\left(x-\frac{\pi}{2}\right)$ or $y=1$.

28. $\tan(x+2y)-\sin x = 1 \Rightarrow \sec^2(x+2y)\left(1+2y'\right)-\cos x = 0$.

At $\left(0,\frac{\pi}{8}\right)$, $2\left(1+2y'\right)-1=0 \Rightarrow y' = -\frac{1}{4}$.

30. $x^3 - y^3 = 8 \Rightarrow 3x^2 - 3y^2y' = 0 \Rightarrow y' = \dfrac{x^2}{y^2}$.

Differentiating both sides of the next-to-last expression yields $6x - 6y\left(y'\right)^2 - 3y^2y'' = 0$

$$\Rightarrow \quad y'' = \frac{2\left[x - y\left(y'\right)^2\right]}{y^2} = \frac{2\left[x - y\left(x^2/y^2\right)^2\right]}{y^2} = \frac{2x\left(y^3 - x^3\right)}{y^5}$$

32. $\tan y - xy = 0 \Rightarrow \left(\sec^2 y\right)y' - y - xy' = 0 \Rightarrow y' = \dfrac{y}{\sec^2 y - x}$. Differentiating both sides of the next-to-last expression yields $\left(2\sec^2 y \tan y\right)\left(y'\right)^2 + \left(\sec^2 y\right)y'' - y' - y' - xy'' = 0 \Rightarrow$

$$y'' = \frac{2y'\left[1 - \left(\sec^2 y \tan y\right)y'\right]}{\sec^2 y - x} = \frac{2\left(\frac{y}{\sec^2 y - x}\right)\left(1 - \frac{y\sec^2 y \tan y}{\sec^2 y - x}\right)}{\sec^2 y - x} = \frac{2y\left(\sec^2 y - x - y\sec^2 y \tan y\right)}{\left(\sec^2 y - x\right)^3}$$

2.9

22. For $\sqrt{63.8}$, let $f(x) = \sqrt{x}$ and $a = 64$. Then $f'(x) = \dfrac{1}{2\sqrt{x}}$, so

$L(x) = f(64) + f'(64)(x - 64) = \sqrt{64} + \dfrac{1}{2\sqrt{64}}(x - 64) = \frac{1}{16}x + 4$.

Thus, $\sqrt{63.8} \approx L(63.8) = \frac{1}{16}(63.8) + 4 = 7.9875$.

24. For $\sin 0.1$, let $f(x) = \sin x$ and $a = 0$. Then $f'(x) = \cos x$,

so $L(x) = f(0) + f'(0)(x - 0) = x$. Thus, $\sin 0.1 \approx L(0.1) = 0.1$.

Chap 2 Review

8. $f'(x) = 6x^2 + 2x$, so $f'(1) = 8$.

12. $$f'(x) = \frac{d}{dx}\left(\frac{x+1}{x-1}\right) = \frac{(x-1)(1)-(x+1)(1)}{(x-1)^2} = -\frac{2}{(x-1)^2}$$

16. $$u' = \frac{d}{dt}\left(\frac{t^2}{1-t^{1/2}}\right) = \frac{\left(1-t^{1/2}\right)(2t) - t^2\left(-\frac{1}{2}t^{-1/2}\right)}{\left(1-t^{1/2}\right)^2} = \frac{2t - 2t^{3/2} + \frac{1}{2}t^{3/2}}{\left(1-t^{1/2}\right)^2} = \frac{t\left(4-3\sqrt{t}\right)}{2\left(1-\sqrt{t}\right)^2}$$

18. $$f'(x) = \frac{d}{dx}(x\tan x + \sec x) = \tan x + x\sec^2 x + \sec x \tan x$$

20. $$y' = \frac{d}{dx}\left(\frac{1-\sin x}{1+\sin x}\right) = \frac{(1+\sin x)(-\cos x)-(1-\sin x)(\cos x)}{(1+\sin x)^2} = -\frac{2\cos x}{(1+\sin x)^2}$$

40. $$f'\left(\tfrac{\pi}{4}\right) = \cos(\cos x)\tfrac{d}{dx}(\cos x)\Big|_{\pi/4} = -(\sin x)\cos(\cos x)|_{\pi/4}$$

$$= -\left(\sin\tfrac{\pi}{4}\right)\cos\left(\cos\tfrac{\pi}{4}\right) = -\tfrac{\sqrt{2}}{2}\cos\tfrac{\sqrt{2}}{2} \approx -0.538$$

52. $f(x) = x\tan x \Rightarrow f'(x) = \tan x + x\sec^2 x$

$$\Rightarrow f''(x) = \sec^2 x + \sec^2 x + 2x\sec^2 x \tan x = 2\left(\sec^2 x\right)(1 + x\tan x)$$

$$\Rightarrow f''\left(\tfrac{\pi}{4}\right) = 2\left(\sqrt{2}\right)^2\left(1+\tfrac{\pi}{4}\right) = 4 + \pi$$

54. $$h(x) = \frac{f(x)}{g(x)} \Rightarrow h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2},$$

so $$h'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2} = \frac{(2)(-1)-(3)(4)}{(2)^2} = -\frac{7}{2}.$$

60. $xy^{1/2} + yx^{1/2} - 1 = 0 \Rightarrow x\left(\frac{1}{2}y^{-1/2}\frac{dy}{dx}\right) + y^{1/2} + y\left(\frac{1}{2}x^{-1/2}\right) + \frac{dy}{dx}x^{1/2} = 0$

$$\Rightarrow \left(\frac{x}{y^{1/2}} + 2x^{1/2}\right)\frac{dy}{dx} = -\left(2y^{1/2} + \frac{y}{x^{1/2}}\right)$$

$$\Rightarrow \left(\frac{x + 2x^{1/2}y^{1/2}}{y^{1/2}}\right)\frac{dy}{dx} = -\frac{2x^{1/2}y^{1/2} + y}{x^{1/2}}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{2\sqrt{x}y + y\sqrt{y}}{x\sqrt{x} + 2x\sqrt{y}}$$

66. $\cos^2 x + \sin^2 y = 1 \Rightarrow (2\cos x)(-\sin x) + (2\sin y\cos y)\frac{dy}{dx} = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin x\cos x}{\sin y\cos y}$$

78. Differentiating implicitly, we have $x + \sqrt{xy} + y = 6 \Rightarrow 1 + \frac{1}{2}(xy)^{-1/2}(y + xy') + y' = 0$. If $x = y = 2$, then we have $1 + \frac{1}{2}\left(\frac{1}{2}\right)(2 + 2y') + y' = 0 \Rightarrow y' = -1$, so an equation of the tangent line is $y - 2 = -1(x - 2) \Leftrightarrow y = -x + 4$ and an equation of the normal line is $y - 2 = 1(x - 2) \Leftrightarrow y = x$.

80. $\sin 2x + \cos 2y = 1 \Rightarrow 2\cos 2x - (2\sin 2y)\, y' = 0 \Rightarrow y' = \frac{\cos 2x}{\sin 2y} \Rightarrow$

$$y'' = \frac{(\sin 2y)(-2\sin 2x) - (\cos 2x)(2\cos 2y)\, y'}{(\sin 2y)^2} = -\frac{2\sin 2x\sin^2 2y + 2\cos^2 2x\cos 2y}{\sin^3 2y}$$