

## 2.7 Implicit Differentiation

$$4. x^2y + 2xy^2 - x + 3 = 0 \Rightarrow 2xy + x^2y' + 2y^2 + 4xyy' - 1 = 0 \Rightarrow (x^2 + 4xy)y' = 1 - 2xy - 2y^2 \Rightarrow y' = \frac{1 - 2xy - 2y^2}{x(x + 4y)}$$

$$12. (2x^2 + 3y^2)^{5/2} = x \Rightarrow (2x^2 + 3y^2) = x^{2/5} \Rightarrow 4x + 6yy' = \frac{2}{5}x^{-3/5} \Rightarrow 6yy' = \frac{2}{5x^{3/5}} - 4x = \frac{2(1 - 10x^{8/5})}{5x^{3/5}} \Rightarrow y' = \frac{1 - 10x^{8/5}}{15x^{3/5}y}$$

$$16. x + y^2 = \cos xy \Rightarrow 1 + 2yy' = (-\sin xy)(y + xy') \Rightarrow (2y + x \sin xy)y' = -y \sin xy - 1 \Rightarrow y' = -\frac{y \sin xy + 1}{2y + x \sin xy}$$

$$22. x^2y + y^3 = 2 \Rightarrow 2xy + x^2y' + 3y^2y' = 0 \Rightarrow y' = -\frac{2xy}{x^2 + 3y^2}, \text{ so } y'|_{(-1,1)} = -\frac{2(-1)(1)}{1+3} = \frac{1}{2}. \text{ An equation of the tangent line is } y - 1 = \frac{1}{2}(x + 1) \text{ or } y = \frac{1}{2}x + \frac{3}{2}.$$

$$24. y = \sin xy \Rightarrow y' = (\cos xy)(y + xy') \Rightarrow y' = \frac{y \cos xy}{1 - x \cos xy} \Rightarrow y'|_{(\pi/2, 1)} = 0. \text{ An equation of the tangent line is } y - 1 = 0 \text{ (} x - \frac{\pi}{2} \text{) or } y = 1.$$

$$28. \tan(x + 2y) - \sin x = 1 \Rightarrow \sec^2(x + 2y)(1 + 2y') - \cos x = 0. \text{ At } (0, \frac{\pi}{8}), 2(1 + 2y') - 1 = 0 \Rightarrow y' = -\frac{1}{4}.$$

$$30. x^3 - y^3 = 8 \Rightarrow 3x^2 - 3y^2y' = 0 \Rightarrow y' = \frac{x^2}{y^2}. \text{ Differentiating both sides of the next-to-last expression yields}$$

$$6x - 6y(y')^2 - 3y^2y'' = 0 \Rightarrow y'' = \frac{2[x - y(y')^2]}{y^2} = \frac{2[x - y(x^2/y^2)^2]}{y^2} = \frac{2x(y^3 - x^3)}{y^5}$$

$$32. \tan y - xy = 0 \Rightarrow (\sec^2 y)y' - y - xy' = 0 \Rightarrow y' = \frac{y}{\sec^2 y - x}. \text{ Differentiating both sides}$$

$$\text{of the next-to-last expression yields } (2 \sec^2 y \tan y)(y')^2 + (\sec^2 y)y'' - y' - y' - xy'' = 0 \Rightarrow$$

$$y'' = \frac{2y'[1 - (\sec^2 y \tan y)y']}{\sec^2 y - x} = \frac{2\left(\frac{y}{\sec^2 y - x}\right)\left(1 - \frac{y \sec^2 y \tan y}{\sec^2 y - x}\right)}{\sec^2 y - x} = \frac{2y(\sec^2 y - x - y \sec^2 y \tan y)}{(\sec^2 y - x)^3}$$

## 2.9 Differentials and Linear Approximations

22. For  $\sqrt{63.8}$ , let  $f(x) = \sqrt{x}$  and  $a = 64$ . Then  $f'(x) = \frac{1}{2\sqrt{x}}$ , so

$$L(x) = f(64) + f'(64)(x - 64) = \sqrt{64} + \frac{1}{2\sqrt{64}}(x - 64) = \frac{1}{16}x + 4. \text{ Thus, } \sqrt{63.8} \approx L(63.8) = \frac{1}{16}(63.8) + 4 = 7.9875.$$

24. For  $\sin 0.1$ , let  $f(x) = \sin x$  and  $a = 0$ . Then  $f'(x) = \cos x$ , so  $L(x) = f(0) + f'(0)(x - 0) = x$ . Thus,  $\sin 0.1 \approx L(0.1) = 0.1$ .

## Chapter 2 Review

8.  $f'(x) = 6x^2 + 2x$ , so  $f'(1) = 8$ .

$$12. f'(x) = \frac{d}{dx} \left( \frac{x+1}{x-1} \right) = \frac{(x-1)(1) - (x+1)(1)}{(x-1)^2} = -\frac{2}{(x-1)^2}$$

$$16. u' = \frac{d}{dt} \left( \frac{t^2}{1-t^{1/2}} \right) = \frac{(1-t^{1/2})(2t) - t^2(-\frac{1}{2}t^{-1/2})}{(1-t^{1/2})^2} = \frac{2t - 2t^{3/2} + \frac{1}{2}t^{3/2}}{(1-t^{1/2})^2} = \frac{t(4-3\sqrt{t})}{2(1-\sqrt{t})^2}$$

$$18. f'(x) = \frac{d}{dx} (x \tan x + \sec x) = \tan x + x \sec^2 x + \sec x \tan x$$

$$20. y' = \frac{d}{dx} \left( \frac{1-\sin x}{1+\sin x} \right) = \frac{(1+\sin x)(-\cos x) - (1-\sin x)(\cos x)}{(1+\sin x)^2} = -\frac{2\cos x}{(1+\sin x)^2}$$

$$40. f' \left( \frac{\pi}{4} \right) = \cos(\cos x) \frac{d}{dx} (\cos x) \Big|_{\pi/4} = -(\sin x) \cos(\cos x) \Big|_{\pi/4} = -\left( \sin \frac{\pi}{4} \right) \cos \left( \cos \frac{\pi}{4} \right) = -\frac{\sqrt{2}}{2} \cos \frac{\sqrt{2}}{2} \approx -0.538$$

$$52. f(x) = x \tan x \Rightarrow f'(x) = \tan x + x \sec^2 x \Rightarrow f''(x) = \sec^2 x + \sec^2 x + 2x \sec^2 x \tan x = 2(\sec^2 x)(1 + x \tan x) \Rightarrow f'' \left( \frac{\pi}{4} \right) = 2(\sqrt{2})^2 \left( 1 + \frac{\pi}{4} \right) = 4 + \pi$$

$$54. h(x) = \frac{f(x)}{g(x)} \Rightarrow h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}, \text{ so } h'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2} = \frac{(2)(-1) - (3)(4)}{(2)^2} = -\frac{7}{2}.$$

$$60. \quad xy^{1/2} + yx^{1/2} - 1 = 0 \Rightarrow x \left( \frac{1}{2}y^{-1/2} \frac{dy}{dx} \right) + y^{1/2} + y \left( \frac{1}{2}x^{-1/2} \right) + \frac{dy}{dx}x^{1/2} = 0 \Rightarrow$$

$$\left( \frac{x}{y^{1/2}} + 2x^{1/2} \right) \frac{dy}{dx} = - \left( 2y^{1/2} + \frac{y}{x^{1/2}} \right) \Rightarrow \left( \frac{x + 2x^{1/2}y^{1/2}}{y^{1/2}} \right) \frac{dy}{dx} = - \frac{2x^{1/2}y^{1/2} + y}{x^{1/2}} \Rightarrow \frac{dy}{dx} = - \frac{2\sqrt{xy} + y\sqrt{y}}{x\sqrt{x} + 2x\sqrt{y}}$$

$$66. \quad \cos^2 x + \sin^2 y = 1 \Rightarrow (2 \cos x) (-\sin x) + (2 \sin y \cos y) \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{\sin x \cos x}{\sin y \cos y}$$

78. Differentiating implicitly, we have  $x + \sqrt{xy} + y = 6 \Rightarrow 1 + \frac{1}{2}(xy)^{-1/2}(y + xy') + y' = 0$ . If  $x = y = 2$ , then we have  $1 + \frac{1}{2} \left( \frac{1}{2} \right) (2 + 2y') + y' = 0 \Rightarrow y' = -1$ , so an equation of the tangent line is  $y - 2 = -1(x - 2) \Leftrightarrow y = -x + 4$  and an equation of the normal line is  $y - 2 = 1(x - 2) \Leftrightarrow y = x$ .

$$80. \quad \sin 2x + \cos 2y = 1 \Rightarrow 2 \cos 2x - (2 \sin 2y) y' = 0 \Rightarrow y' = \frac{\cos 2x}{\sin 2y} \Rightarrow$$

$$y'' = \frac{(\sin 2y)(-2 \sin 2x) - (\cos 2x)(2 \cos 2y) y'}{(\sin 2y)^2} = - \frac{2 \sin 2x \sin^2 2y + 2 \cos^2 2x \cos 2y}{\sin^3 2y}$$