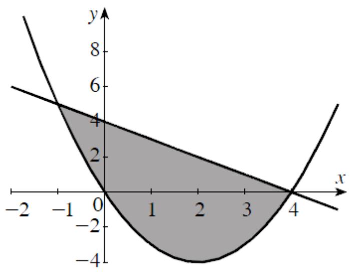
12. Solve $x^2 - 4x = -x + 4 \Leftrightarrow x^2 - 3x - 4 = (x - 4)(x + 1) = 0$, giving $(-1, 5)$ and $(4, 0)$ as the points of intersection. Thus,

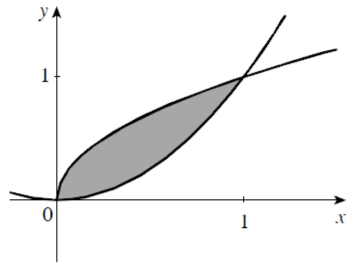
$$A = \int_{-1}^{4} \left[(-x + 4) - \left(x^2 - 4x\right)\right] dx = \int_{-1}^{4} \left(-x^2 + 3x + 4\right) dx$$

$$= -\tfrac{1}{3}x^3 + \tfrac{3}{2}x^2 + 4x\Big|_{-1}^{4} = \tfrac{125}{6}$$

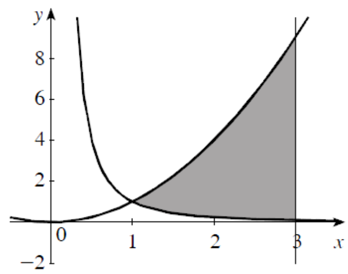


17. Solve $\sqrt{x} = x^2 \Leftrightarrow x = x^4 \Leftrightarrow x\left(x^3 - 1\right) = 0$, giving $(0, 0)$ and $(1, 1)$ as the points of intersection. Thus,

$$A = \int_0^1 \left(\sqrt{x} - x^2\right) dx = \tfrac{2}{3}x^{3/2} - \tfrac{1}{3}x^3\Big|_0^1 = \tfrac{1}{3}.$$



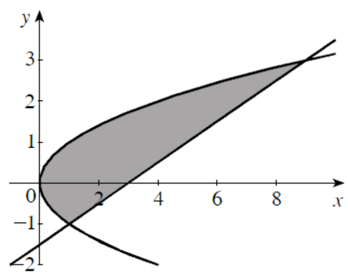
21. $A = \int_1^3 \left(x^2 - \frac{1}{x^2}\right) dx = \frac{1}{3}x^3 + \frac{1}{x}\Big|_1^3 = \left(9 + \frac{1}{3}\right) - \left(\frac{1}{3} + 1\right) = 8$



28. Solve $y^2 = 2y + 3 \Leftrightarrow y^2 - 2y - 3 = (y - 3)(y + 1) = 0$, giving $(1, -1)$ and $(9, 3)$ as the points of intersection. Thus,

$$A = \int_{-1}^{3} \left[(2y + 3) - y^2\right] dy = \int_{-1}^{3} \left(-y^2 + 2y + 3\right) dy$$

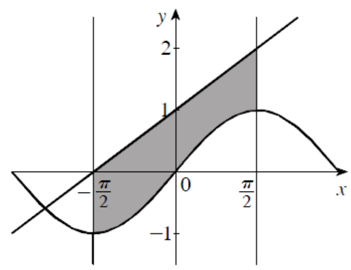
$$= -\tfrac{1}{3}y^3 + y^2 + 3y\Big|_{-1}^{3} = \tfrac{32}{3}$$



32. $A = \int_{-\pi/2}^{\pi/2} \left[\left(\frac{2}{\pi}x + 1\right) - \sin x\right] dx$

$$= 2\int_0^{\pi/2} 1\, dx = \pi$$

because $f(x) = \frac{2}{\pi}x - \sin x$ is odd.



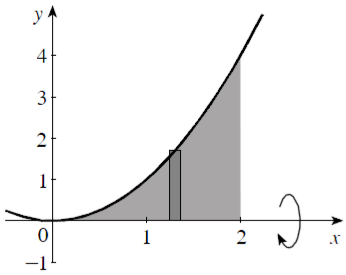
5.2 Volumes: Disks, Washers, and Cross-Sections

7. $V = \pi \int_0^1 \left[\left(y^{1/3}\right)^2 - \left(y^{1/2}\right)^2 \right] dy = \pi \int_0^1 \left(y^{2/3} - y\right) dy = \pi \left(\frac{3}{5} y^{5/3} - \frac{1}{2} y^2 \right) \Big|_0^1 = \frac{\pi}{10}$

11. $V = \pi \int_0^1 \left[\left(\frac{3}{2} - x^3\right)^2 - \left(\frac{3}{2} - x\right)^2 \right] dx = \pi \int_0^1 \left(x^6 - 3x^3 - x^2 + 3x\right) dx = \pi \left(\frac{1}{7}x^7 - \frac{3}{4}x^4 - \frac{1}{3}x^3 + \frac{3}{2}x^2 \right) \Big|_0^1 = \frac{47\pi}{84}$

13. $V = \pi \int_0^2 y^2\, dx = \pi \int_0^2 \left(x^2\right)^2 dx = \pi \int_0^2 x^4\, dx$

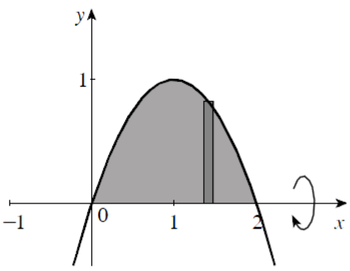
$= \pi \left(\frac{1}{5} x^5 \right) \Big|_0^2 = \frac{32\pi}{5}$



15. $V = \pi \int_0^2 y^2\, dx = \pi \int_0^2 \left(-x^2 + 2x\right)^2 dx$

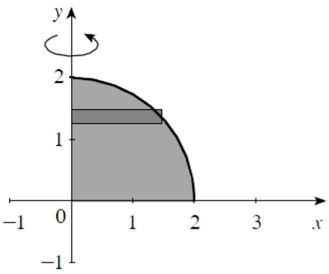
$= \pi \int_0^2 \left(x^4 - 4x^3 + 4x^2\right) dx$

$= \pi \left(\frac{1}{5}x^5 - x^4 + \frac{4}{3}x^3 \right) \Big|_0^2 = \frac{16\pi}{15}$



19. $V = \pi \int_0^2 x^2\, dy = \pi \int_0^2 \left(\sqrt{4 - y^2}\right)^2 dy$

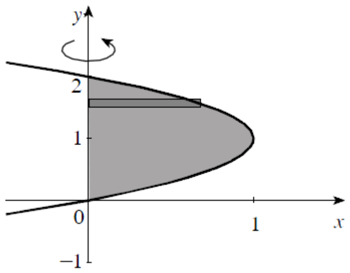
$= \pi \int_0^2 \left(4 - y^2\right) dy = \pi \left(4y - \frac{1}{3}y^3\right) \Big|_0^2 = \frac{16\pi}{3}$



20. $V = \pi \int_0^2 x^2\, dy = \pi \int_0^2 \left(-y^2 + 2y\right)^2\, dy$

$= \pi \int_0^2 \left(y^4 - 4y^3 + 4y^2\right) dy$

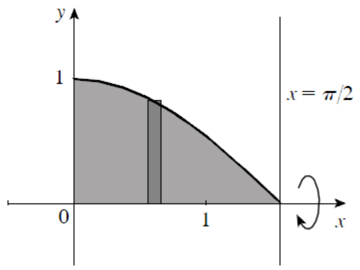
$= \pi \left(\frac{1}{5}y^5 - y^4 + \frac{4}{3}y^3\right)\Big|_0^2 = \frac{16\pi}{15}$



25. $V = \pi \int_0^{\pi/2} y^2\, dx = \pi \int_0^{\pi/2} \cos^2 x\, dx$

$= \frac{\pi}{2} \int_0^{\pi/2} (1 + \cos 2x)\, dx$

$= \frac{\pi}{2}\left(x + \frac{1}{2}\sin 2x\right)\Big|_0^{\pi/2} = \frac{\pi^2}{4}$



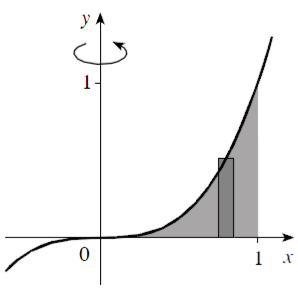
5.3 Volumes Using Cylindrical Shells

4. $V = 2\pi \int_0^2 y\left(4 - y^2\right) dy = 2\pi \int_0^2 \left(4y - y^3\right) dy = 2\pi\left(2y^2 - \frac{1}{4}y^4\right)\Big|_0^2 = 8\pi$

6. $V = 2\pi \int_0^2 (3 - x)\left[\left(\frac{1}{2}x^2 + 2\right) - x^2\right] dx = 2\pi \int_0^2 \left(\frac{1}{2}x^3 - \frac{3}{2}x^2 - 2x + 6\right) dx = 2\pi\left(\frac{1}{8}x^4 - \frac{1}{2}x^3 - x^2 + 6x\right)\Big|_0^2 = 12\pi$

8. $V = 2\pi \int_0^1 xy\, dx = 2\pi \int_0^1 x\left(x^3\right) dx$

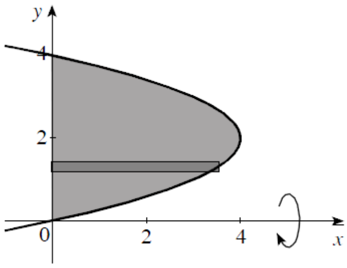
$= 2\pi \int_0^1 x^4\, dx = 2\pi\left(\frac{1}{5}x^5\right)\Big|_0^1 = \frac{2\pi}{5}$



12. $V = 2\pi \int_0^4 yx\,dy = 2\pi \int_0^4 y\left(-y^2 + 4y\right) dy$

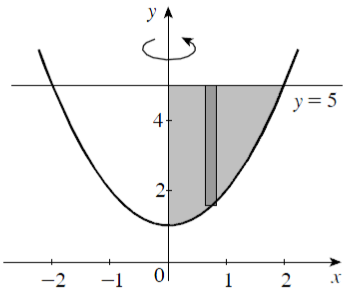
$= 2\pi \int_0^4 \left(-y^3 + 4y^2\right) dy = 2\pi \left(-\frac{1}{4}y^4 + \frac{4}{3}y^3\right)\Big|_0^4$

$= \frac{128\pi}{3}$



15. $V = 2\pi \int_0^2 x\left[5 - \left(x^2 + 1\right)\right] dx$

$= 2\pi \int_0^2 \left(-x^3 + 4x\right) dx = 2\pi \left(-\frac{1}{4}x^4 + 2x^2\right)\Big|_0^2 = 8\pi$



16. To find the points of intersection of the two graphs, we solve $\frac{1}{2}x^2 = x \Leftrightarrow x\,(x - 2) = 0$, giving the points $(0, 0)$ and $(2, 2)$.

$V = 2\pi \int_0^2 x\left(x - \frac{1}{2}x^2\right) dx = 2\pi \int_0^2 \left(x^2 - \frac{1}{2}x^3\right) dx$

$= 2\pi \left(\frac{1}{3}x^3 - \frac{1}{8}x^4\right)\Big|_0^2 = \frac{4\pi}{3}$