2.2 Basic Rules of Differentiation

12. $g'(x) = \frac{d}{dx}\left(-\frac{1}{3}x^2 + \sqrt{2}x\right) = -\frac{2}{3}x + \sqrt{2}$

22. $f'(x) = \frac{d}{dx}\left(5x^{4/3} - \frac{2}{3}x^{3/2} + x^2 - 3x + 1\right) = \frac{20}{3}x^{1/3} - x^{1/2} + 2x - 3$

24. $f'(x) = \frac{d}{dx}\left[-\frac{1}{3}\left(x^{-3} - x^6\right)\right] = -\frac{1}{3}\left(-3x^{-4} - 6x^5\right) = x^{-4} + 2x^5 = 2x^5 + \frac{1}{x^4}$

26. $f'(x) = \frac{d}{dx}\left(5x^{-3} - 2x^{-2} - x^{-1} + 200\right) = -15x^{-4} + 4x^{-3} + x^{-2} = -\frac{15}{x^4} + \frac{4}{x^3} + \frac{1}{x^2}$

32. $f'(u) = \frac{d}{du}\left(u^{-1/2} - 3u^{-1/3}\right) = -\frac{1}{2}u^{-3/2} + u^{-4/3} = -\frac{1}{2u^{3/2}} + \frac{1}{u^{4/3}}$

33. $f'(x) = \frac{d}{dx}\left(2x^3 - 4x\right) = 6x^2 - 4$

a. $f'(-2) = 6(-2)^2 - 4 = 24 - 4 = 20$

b. $f'(0) = 6(0) - 4 = -4$

c. $f'(2) = 6(2)^2 - 4 = 20$

40. $g'(x) = x^2 - x - 1 = -1 \Rightarrow x^2 - x = x(x-1) = 0 \Rightarrow x = 0$ or 1. $g(0) = 1$ and $g(1) = \frac{1}{3} - \frac{1}{2} - 1 + 1 = -\frac{1}{6}$, so the points are $(0, 1)$ and $\left(1, -\frac{1}{6}\right)$.

42. $F'(s) = \frac{d}{ds}\left(2 + \frac{1}{s}\right) = -\frac{1}{s^2} = -\frac{1}{9} \Rightarrow s^2 = 9 \Rightarrow s = \pm 3$. $F(3) = \frac{2(3)+1}{3} = \frac{7}{3}$ and $F(-3) = \frac{2(-3)+1}{-3} = \frac{5}{3}$, so the points are $\left(3, \frac{7}{3}\right)$ and $\left(-3, \frac{5}{3}\right)$.

50. The line $y = 2x$ has slope 2. Also $y = x^2 + c \Rightarrow \frac{dy}{dx} = 2x$, so $2x = 2 \Rightarrow x = 1$. Therefore $y = 2$. Substituting into the second equation gives $2 = 1 + c$, so $c = 1$.

54. $\lim_{h\to 0} \frac{x^5 - 1}{x - 1} = \lim_{h\to 0} \frac{(1+h)^5 - 1}{h} = f'(1)$, where $f(x) = x^5$. $f'(x) = 5x^4$, so $f'(1) = 5$.

58. We use the definition of the derivative to write

$$\lim_{h\to 0}\frac{\dfrac{1}{x+h}+\sqrt{x+h}-\dfrac{1}{x}-\sqrt{x}}{h}=\lim_{h\to 0}\frac{\dfrac{1}{x+h}-\dfrac{1}{x}}{h}+\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}=\frac{d}{dx}\left(\frac{1}{x}\right)+\frac{d}{dx}\left(\sqrt{x}\right)=-\frac{1}{x^2}+\frac{1}{2\sqrt{x}}.$$

72. In order for f to be continuous at a, $\lim_{x\to a^-} f(x) = \lim_{x\to a^-} x^2 = a^2$ must be equal to $\lim_{x\to a^+} f(x) = \lim_{x\to a^+}(Ax+B) = Aa+B$, that is, $aA+B=a^2$. In order for f to be differentiable at a, we must have $\lim_{h\to 0^-}\dfrac{f(a+h)-f(a)}{h} = \lim_{h\to 0^+}\dfrac{f(a+h)-f(a)}{h}$ or $2a = A$. Therefore, $A = 2a$ and $B = a^2-2a^2 = -a^2$.

2.3 The Product and Quotient Rules

4. $f'(x) = (3x+1)\dfrac{d}{dx}\left(x^2-2\right)+\left(x^2-2\right)\dfrac{d}{dx}(3x+1) = (3x+1)(2x)+\left(x^2-2\right)(3) = 6x^2+2x+3x^2-6$
$= 9x^2+2x-6$

6.
$$\begin{aligned} f'(w) &= \left(w^3-w^2+w-1\right)\frac{d}{dw}\left(w^3+2\right)+\left(w^3+2\right)\frac{d}{dw}\left(w^3-w^2+w-1\right)\\ &= \left(w^3-w^2+w-1\right)\left(3w^2\right)+\left(w^3+2\right)\left(3w^2-2w+1\right)\\ &= 3w^5-3w^4+3w^3-3w^2+3w^5-2w^4+w^3+6w^2-4w+2 = 6w^5-5w^4+4w^3+3w^2-4w+2 \end{aligned}$$

10. $$P'(t) = \frac{(3-2t)\dfrac{d}{dt}(2-t)-(2-t)\dfrac{d}{dt}(3-2t)}{(3-2t)^2} = \frac{(3-2t)(-1)-(2-t)(-2)}{(3-2t)^2} = \frac{-3+2t+4-2t}{(3-2t)^2} = \frac{1}{(3-2t)^2}$$

12. $$f'(s) = \frac{(s+1)\dfrac{d}{ds}\left(s^2-4\right)-\left(s^2-4\right)\dfrac{d}{ds}(s+1)}{(s+1)^2} = \frac{(s+1)(2s)-\left(s^2-4\right)(1)}{(s+1)^2} = \frac{2s^2+2s-s^2+4}{(s+1)^2}$$
$$= \frac{s^2+2s+4}{(s+1)^2}$$

19. $$f'(x) = \frac{\left(x^2+1\right)\dfrac{d}{dx}\left(2x^{1/2}\right)-\left(2x^{1/2}\right)\dfrac{d}{dx}\left(x^2+1\right)}{\left(x^2+1\right)^2} = \frac{\left(x^2+1\right)\left(x^{-1/2}\right)-\left(2x^{1/2}\right)(2x)}{\left(x^2+1\right)^2}$$
$$= \frac{x^{3/2}+x^{-1/2}-4x^{3/2}}{\left(x^2+1\right)^2} = \frac{-3x^{3/2}+x^{-1/2}}{\left(x^2+1\right)^2} = \frac{x^{-1/2}\left(-3x^2+1\right)}{\left(x^2+1\right)^2} = \frac{-3x^2+1}{\sqrt{x}\left(x^2+1\right)^2}$$

24. $f'(r) = \frac{d}{dr}\left[\frac{(2r+1)(r-3)}{3r+1}\right] = \frac{d}{dr}\left(\frac{2r^2-5r-3}{3r+1}\right) = \frac{(3r+1)\frac{d}{dr}\left(2r^2-5r-3\right) - \left(2r^2-5r-3\right)\frac{d}{dr}(3r+1)}{(3r+1)^2}$

$= \frac{(3r+1)(4r-5) - \left(2r^2-5r-3\right)(3)}{(3r+1)^2} = \frac{\left(12r^2-15r+4r-5\right) - \left(6r^2-15r-9\right)}{(3r+1)^2} = \frac{6r^2+4r+4}{(3r+1)^2}$

$= \frac{2\left(3r^2+2r+2\right)}{(3r+1)^2}$

38. $f(x) = \frac{x}{x^2+1} \Rightarrow f'(x) = \frac{\left(x^2+1\right)(1) - x(2x)}{\left(x^2+1\right)^2} = \frac{1-x^2}{\left(x^2+1\right)^2}$. At a point where the tangent line is horizontal, we have $f'(x) = 0 \Rightarrow 1 - x^2 = 0$, giving $x = \pm 1$. Therefore, the required points are $\left(-1, -\frac{1}{2}\right)$ and $\left(1, \frac{1}{2}\right)$.

50. $h'(x) = \frac{[f(x)-g(x)]\frac{d}{dx}[f(x)g(x)] - f(x)g(x)\frac{d}{dx}[f(x)-g(x)]}{[f(x)-g(x)]^2}$

$= \frac{[f(x)-g(x)][f(x)g'(x)+g(x)f'(x)] - f(x)g(x)[f'(x)-g'(x)]}{[f(x)-g(x)]^2} \Rightarrow$

$h'(1) = \frac{[f(1)-g(1)][f(1)g'(1)+g(1)f'(1)] - f(1)g(1)[f'(1)-g'(1)]}{[f(1)-g(1)]^2}$

$= \frac{[2-(-2)][(2)(3)+(-2)(-1)] - (2)(-2)(-1-3)}{[2-(-2)]^2} = 1$

52. Take $f(x) = (x+1)^2$. Then $f'(1) = \lim_{x\to 1}\frac{f(x)-f(1)}{x-1} = \lim_{x\to 1}\frac{(x+1)^2-4}{x-1}$. But $f'(x) = \frac{d}{dx}(x+1)^2 = \frac{d}{dx}\left(x^2+2x+1\right) = 2x+2$. Therefore, $\lim_{x\to 1}\frac{(x+1)^2-4}{x-1} = f'(1) = (2x+2)|_{x=1} = 4$.

55. $f(x) = x^{-1} + 3x^{-2} \Rightarrow f'(x) = -x^{-2} - 6x^{-3} \Rightarrow f''(x) = 2x^{-3} + 18x^{-4} = \frac{2}{x^3} + \frac{18}{x^4}$

58. $y = x^2\left(x + \frac{1}{x}\right) = x^3 + x \Rightarrow y' = 3x^2 + 1 \Rightarrow y'' = 6x$

61. **a.** $f(x) = 4x^3 - 2x^2 + 3 \Rightarrow f'(x) = 12x^2 - 4x \Rightarrow f''(x) = 24x - 4$, so $f''(2) = 24(2) - 4 = 44$.

b. $y = 2x^3 - \frac{1}{x} = 2x^3 - x^{-1} \Rightarrow y' = 6x^2 + x^{-2} \Rightarrow y'' = 12x - 2x^{-3}$, so $y''|_{x=1} = 12 - 2 = 10$.