Calculus

2013/10/22

考試注意事項：

1. 答案紙直行對折，兩直欄書寫作答。
2. 無清楚計算過程，不予計分。

試題：

1. (15%) Find the extreme values of the function

$$f(x) = 2\sin x + \sin 2x \quad \text{on} \quad [0, \frac{3\pi}{2}].$$

2. (15%) Use the Mean Value Theorem to prove that

$$|\sin a - \sin b| \leq |a - b| \quad \text{for all real numbers } a \text{ and } b.$$

3. (15%) Use the First Derivative Test to find the relative extrema of

$$f(x) = 15x^{2/3} - 3x^{5/3}.$$

4. (15%) Use the Second Derivative Test to find the relative extrema of

$$h(x) = 2x^3 + 3x^2 - 12x - 2.$$

5. (10%) Find an equation of the tangent line to the bifolium

$$4x^4 + 8x^2y^2 - 25x^2y + 4y^4 = 0$$

at the point $(2, 1)$.

6. (10%) Find $\frac{dy}{dx}$ if $y = \tan^3(3x^2 + 1)$.

7. (10%) Suppose that f and g are functions that are differentiable at $x = 1$ and that $f(1) = 2$, $f'(1) = -1$, $g(1) = -2$, and $g'(1) = 3$. Find $h'(1)$ with

$$h(x) = \frac{xf(x)}{x + g(x)}.$$

8. (5%,5%) Find the indicated limits:

(a) $\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$

(b) $\lim_{x \to 0} \frac{\sin 2x}{3x}$