#1

$$f(x)=\begin{cases} x\sin(1/x) & \text{if } x\neq 0\\ 0 & \text{if } x=0\end{cases}$$

Because $|\sin(1/x)|\leq 1$, we have $0\leq |x\sin(1/x)|\leq |x|$. Since $\lim_{x\to 0}|x|=0$, the Squeeze Theorem implies that $\lim_{x\to 0} x\sin(1/x)=0=f(0)$, and so f is continuous at 0.

To show that f is not differentiable at 0, we compute $f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{h\sin(1/h)}{h}=\lim_{h\to 0}\sin(1/h)$ which does not exist. Therefore, f is not differentiable at 0.

#2

$$h'(x)=\frac{[f(x)-g(x)]\frac{d}{dx}[f(x)g(x)]-f(x)g(x)\frac{d}{dx}[f(x)-g(x)]}{[f(x)-g(x)]^2}$$

$$=\frac{[f(x)-g(x)][f(x)g'(x)+g(x)f'(x)]-f(x)g(x)[f'(x)-g'(x)]}{[f(x)-g(x)]^2}\Rightarrow$$

$$h'(1)=\frac{[f(1)-g(1)][f(1)g'(1)+g(1)f'(1)]-f(1)g(1)[f'(1)-g'(1)]}{[f(1)-g(1)]^2}$$

$$=\frac{[2-(-2)][(2)(3)+(-2)(-1)]-(2)(-2)(-1-3)}{[2-(-2)]^2}=1$$

#3

$$\frac{dx}{dy}=\frac{d}{dx}[tan^3(3x^2+1)]=\frac{d}{dx}[tan(3x^2+1)]^3$$

$$=3[tan(3x^2+1)]^2\cdot\frac{d}{dx}[tan(3x^2+1)]$$

$$=3tan^2(3x^2+1)\cdot sec^2(3x^2+1)\cdot\frac{d}{dx}(3x^2+1)$$

$$=3tan^2(3x^2+1)\cdot sec^2(3x^2+1)\cdot 6x$$

$$=18xtan^2(3x^2+1)sec^2(3x^2+1)$$

#4

$$x+y^2=\cos xy\Rightarrow 1+2yy'=(-\sin xy)(y+xy')$$

$$\Rightarrow(2y+x\sin xy)y'=-y\sin xy-1\Rightarrow y'=-\frac{y\sin xy+1}{2y+x\sin xy}$$

#5(a)

Using Law 11, we obtain

$$\lim_{x\to 1}\frac{4x^2-3x+1}{2x-4}=\frac{4(3)^2-3(3)+1}{2(3)-4}=\frac{28}{2}=14$$

#5(b)

$$\lim_{x\to 5}\frac{5-x}{x^2-25}=\lim_{x\to 5}\frac{(-1)(x-5)}{(x+5)(x-5)}=(-1)\lim_{x\to 5}\frac{1}{x+5}=-\frac{1}{10}$$

#6

Since $\lim_{x\to 0}\frac{\sin 2x}{x}=\lim_{x\to 0}\left(2\cdot\frac{\sin 2x}{2x}\right)=2$, we require that $f(0)=2$. So we take $c=2$.

#7

The slope of the tangent line at any point (x,y) on the graph of y = xsinx is given by

$$\frac{dx}{dy}=\frac{d}{dx}[x(sinx)]=x\frac{d}{dx}(sinx)+(sinx)\frac{d}{dx}(x)=xcosx+sinx$$

In particular, the slope of the tangent line at the point where x = $\pi/2$ is

$$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{2}}=(xcosx+sinx)|_{x=\frac{\pi}{2}}$$

$$=\frac{\pi}{2}cos\frac{\pi}{2}+sin\frac{\pi}{2}$$

$$=\frac{\pi}{2}(0)+1=1$$

The y-coordinate of the point of tangency is

$$y|_{x=\frac{\pi}{2}}=xsinx|_{x=\frac{\pi}{2}}$$

$$=\frac{\pi}{2}sin\frac{\pi}{2}=\frac{\pi}{2}$$

Using the point-slope form of an equation of a line, we find that

$$y-\frac{\pi}{2}=x-\frac{\pi}{2}\ or\ y=x$$

#8

For $\sqrt{63.8}$, let $f(x) = \sqrt{x}$ and $a = 64$. Then $f'(x) = \dfrac{1}{2\sqrt{x}}$, so $L(x) = f(64) + f'(64)(x - 64) = \sqrt{64} + \dfrac{1}{2\sqrt{64}}(x - 64) = \frac{1}{16}x + 4$. Thus, $\sqrt{63.8} \approx L(63.8) = \frac{1}{16}(63.8) + 4 = 7.9875$.