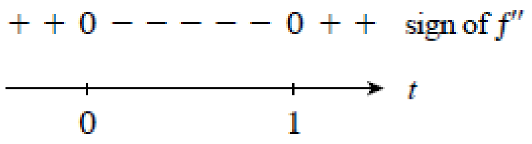
3.4

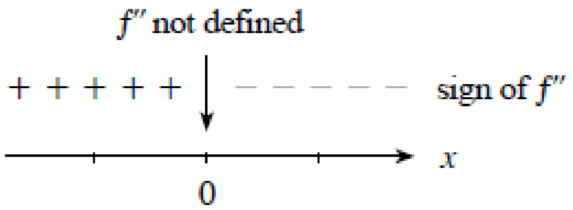
13. $f(t) = t^4 - 2t^3 \Rightarrow f'(t) = 4t^3 - 6t^2 \Rightarrow f''(t) = 12t^2 - 12t = 12t(t-1)$.

The sign diagram of f'' is shown at right. We see that f is concave upward on $(-\infty, 0)$ and $(1, \infty)$ and concave downward on $(0, 1)$. It has points of inflection at $(0, 0)$ and $(1, -1)$.



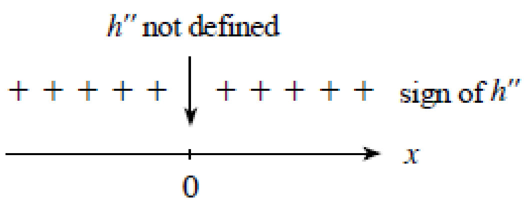
15. $f(x) = 1 + 3x^{1/3} \Rightarrow f'(x) = x^{-2/3} \Rightarrow f''(x) = -\frac{2}{3}x^{-5/3} = -\dfrac{2}{3x^{5/3}}$.

The sign diagram of f'' is shown at right. We see that f is concave upward on $(-\infty, 0)$ and concave downward on $(0, \infty)$. It has an inflection point at $(0, 1)$.



21. $h(x) = x^2 + x^{-2} \Rightarrow h'(x) = 2x - 2x^{-3} \Rightarrow h''(x) = 2 + 6x^{-4} = \dfrac{2\left(x^4+3\right)}{x^4}$.

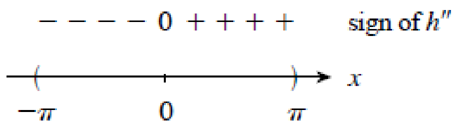
The sign diagram of h'' is shown at right. We see that h is concave upward on $(-\infty, 0)$ and $(0, \infty)$. It has no inflection point.



31. $h(x) = \dfrac{\sin x}{1+\cos x}$, $-\pi < x < \pi \Rightarrow h'(x) = \dfrac{(1+\cos x)\cos x - \sin x(-\sin x)}{(1+\cos x)^2} = \dfrac{1}{1+\cos x}$

$\Rightarrow h''(x) = \dfrac{d}{dx}(1+\cos x)^{-1} = \dfrac{\sin x}{(1+\cos x)^2}$.

The sign diagram of h'' is shown at right. We see that h is concave downward on $(-\pi, 0)$ and concave upward on $(0, \pi)$. It has an inflection point at $(0, 0)$.



37. $h(t) = \frac{1}{3}t^3 - 2t^2 - 5t - 10 \Rightarrow h'(t) = t^2 - 4t - 5 = (t-5)(t+1) = 0 \Rightarrow t = -1$ or 5, the critical numbers of h. $h''(t) = 2t - 4 = 2(t-2)$. We use the SDT: $h''(-1) = -6 < 0$, so h has a relative maximum of $h(-1) = -\frac{22}{3}$; and $h''(5) = 6 > 0$, so h has a relative minimum of $h(5) = -\frac{130}{3}$.

41. $f(t) = 2t + \dfrac{1}{t} \Rightarrow f'(t) = 2 - t^{-2} = \dfrac{2t^2-1}{t^2} = 0 \Rightarrow t = \pm\frac{\sqrt{2}}{2}$, the critical numbers of f. $f''(t) = \dfrac{2}{t^3}$, and we use the SDT: $f''\left(-\frac{\sqrt{2}}{2}\right) < 0$, so f has a relative maximum of $f\left(-\frac{\sqrt{2}}{2}\right) = -2\sqrt{2}$; and $f''\left(\frac{\sqrt{2}}{2}\right) > 0$, so f has a relative minimum of $f\left(\frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$.

45. $f(x) = \sin x + \cos x$, $0 < x < \frac{\pi}{2} \Rightarrow f'(x) = \cos x - \sin x = 0 \Leftrightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}$ in $\left(0, \frac{\pi}{2}\right)$, so this is the only relevant critical number. $f''(x) = -\sin x - \cos x$, so using the SDT, we find that $f''\left(\frac{\pi}{4}\right) = -\sqrt{2} < 0$, implying that f has a relative maximum of $f\left(\frac{\pi}{4}\right) = \sqrt{2}$.

3.5

8. $\displaystyle\lim_{t\to -3^+} \frac{t}{t+3} = -\infty$ since the numerator approaches -3 and the denominator approaches 0 through positive values as $t \to -3$ from the right.

12. $\displaystyle\lim_{t\to 1} \frac{t^3}{(t^2-1)^2} = \infty$ since the numerator approaches 1 and the denominator approaches 0 through positive values as $t \to 1$.

14. $\lim_{x\to -1^+}\left(\frac{1}{x}-\frac{1}{x+1}\right)=-\infty$. As $x\to -1$ from the right,

the first term approaches -1 but the second term approaches ∞.

16. $\lim_{x\to 0^+}\frac{1}{\sin x}=\infty$ since the numerator is positive and the denominator

approaches 0 through positive values as $x\to 0$ from the right.

22. $\lim_{x\to\infty}\frac{2x^2-1}{4x^2+1}=\lim_{x\to\infty}\frac{2-\frac{1}{x^2}}{4+\frac{1}{x^2}}=\frac{1}{2}$

24. $\lim_{x\to -\infty}\frac{2x^3+x^2+3}{x+1}=\lim_{x\to -\infty}\frac{2x^2\left(1+\frac{1}{2x}+\frac{3}{2x^3}\right)}{1+\frac{1}{x}}=\infty$

32. $\lim_{t\to -\infty}\frac{2t^2}{\sqrt{t^4+t^2}}=\lim_{t\to -\infty}\frac{2t^2}{\sqrt{t^4+t^2}}\cdot\frac{\frac{1}{t^2}}{\frac{1}{t^2}}=\lim_{t\to -\infty}\frac{2}{\sqrt{1+\frac{1}{t^2}}}=2$

50. $\lim_{x\to\infty}\frac{x}{x+1}=1$, so $y=1$ is a horizontal asymptote.

$\lim_{x\to -1^+}\frac{x}{x+1}=-\infty$, so $x=-1$ is a vertical asymptote.

52. $\lim_{t\to\infty}\frac{t^2}{t^2-4}=1$, so $y=1$ is a horizontal asymptote.

$\lim_{t\to -2^-}\frac{t^2}{(t+2)(t-2)}=\infty$ and $\lim_{t\to 2^-}\frac{t^2}{(t+2)(t-2)}=-\infty$

so $t=\pm 2$ are vertical asymptotes.

54. $\lim_{x\to\infty} \frac{2-x^2}{x^2+x} = -1$, so $y = -1$ is a horizontal asymptote.

$$\lim_{x\to -1^-} \frac{2-x^2}{x\,(x+1)} = \infty \text{ and } \lim_{x\to 0^+} \frac{2-x^2}{x\,(x+1)} = \infty,$$

so $x = -1$ and $x = 0$ are vertical asymptotes.