

1.1

2. a. $\lim_{x \rightarrow 2^-} f(x) = 2$ 4. a. $\lim_{x \rightarrow 3^-} f(x) = 2$

b. $\lim_{x \rightarrow 2^+} f(x) = -1$ b. $\lim_{x \rightarrow 3^+} f(x) = 2$

c. $\lim_{x \rightarrow 2} f(x)$ does not exist. c. $\lim_{x \rightarrow 3} f(x) = 2$

8. a. True.

b. True. $\lim_{x \rightarrow 0^-} f(x) = 3 = \lim_{x \rightarrow 0^+} f(x)$

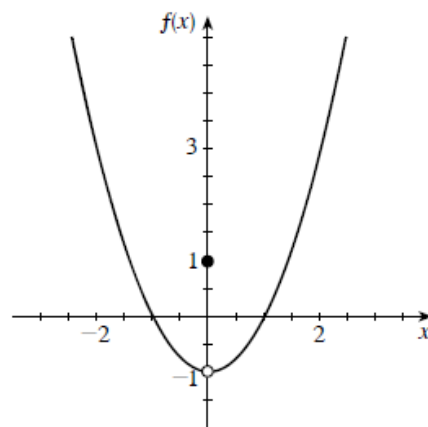
c. True.

d. True.

e. False. Since $\lim_{x \rightarrow 3^-} f(x) = 3$ and $\lim_{x \rightarrow 3^+} f(x) = 3$, $\lim_{x \rightarrow 3} f(x) = 3$.

f. True.

20.

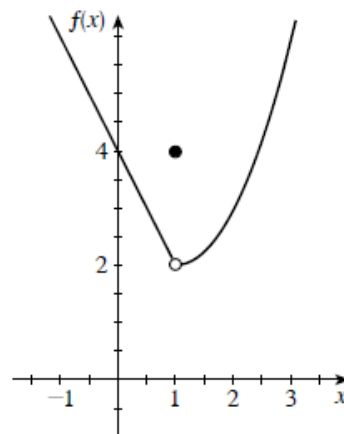


a. $\lim_{x \rightarrow 0^-} f(x) = -1$

b. $\lim_{x \rightarrow 0^+} f(x) = -1$

c. $\lim_{x \rightarrow 0} f(x) = -1$

22.



a. $\lim_{x \rightarrow 1^-} f(x) = 2$

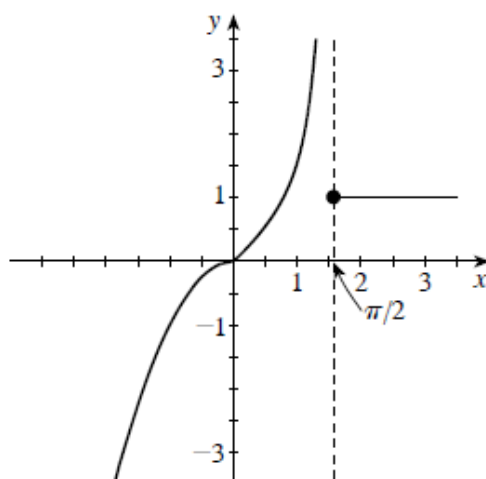
b. $\lim_{x \rightarrow 1^+} f(x) = 2$

c. $\lim_{x \rightarrow 1} f(x) = 2$

$$24. \lim_{x \rightarrow 3^+} \llbracket x \rrbracket = 3$$

$$26. \lim_{x \rightarrow -1} \llbracket x \rrbracket \text{ does not exist.}$$

32. a.



b. The limit of $f(x)$ exists for all values of x in $(-\infty, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \infty)$.

c. The left-hand limit of $f(x)$ exists for all values of x in $(-\infty, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \infty)$.

d. The right-hand limit of $f(x)$ exists for all values of x in $(-\infty, \infty)$

1.2

$$12. \lim_{x \rightarrow -2} (x+3)^2 \sqrt{4x^2 - 8} = 1^2 \sqrt{4(-2)^2 - 8} = 2\sqrt{2}$$

$$22. \lim_{x \rightarrow \pi/4} \frac{\tan^2 x}{1 + \cos x} = \frac{(\tan \frac{\pi}{4})^2}{1 + \cos \frac{\pi}{4}} = \frac{1^2}{1 + \frac{\sqrt{2}}{2}} = \frac{2}{2 + \sqrt{2}} = 2 - \sqrt{2}$$

$$28. \lim_{x \rightarrow a} \frac{\sqrt[3]{f(x)g(x)}}{\sqrt{f(x)g(x)+1}} = \frac{\sqrt[3]{\lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)}}{\sqrt{\lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x) + 1}} = \frac{\sqrt[3]{2 \cdot 4}}{\sqrt{2 \cdot 4 + 1}} = \frac{2}{3}$$

$$42. \lim_{x \rightarrow 5} \frac{5-x}{x^2-25} = \lim_{x \rightarrow 5} \frac{(-1)(x-5)}{(x+5)(x-5)} = (-1) \lim_{x \rightarrow 5} \frac{1}{x+5} = -\frac{1}{10}$$

$$58. \lim_{h \rightarrow 0} \frac{\sqrt{a+h} - \sqrt{a}}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{a+h} - \sqrt{a}}{h} \cdot \frac{\sqrt{a+h} + \sqrt{a}}{\sqrt{a+h} + \sqrt{a}} = \lim_{h \rightarrow 0} \frac{a+h-a}{h(\sqrt{a+h} + \sqrt{a})}$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{a+h} + \sqrt{a}} = \frac{1}{2\sqrt{a}} = \frac{\sqrt{a}}{2a}$$

$$60. \lim_{h \rightarrow 0} \frac{(2+h)^{-1} - 2^{-1}}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h} = \lim_{h \rightarrow 0} \frac{2 - (2+h)}{(2+h)2h} = \lim_{h \rightarrow 0} \frac{-1}{2(2+h)} = -\frac{1}{4}$$

$$66. \lim_{x \rightarrow 0} \frac{\sin 2x}{x} = \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{2x} \cdot \frac{2}{1} \right) = 2 \cdot \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 2 \cdot 1 = 2.$$

We have made the substitution $\theta = 2x$ at the third step.

$$68. \lim_{x \rightarrow 0} \frac{\tan 2x}{3x} = \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{\cos 2x} \cdot \frac{1}{3x} \right) = \left(\lim_{x \rightarrow 0} \frac{1}{\cos 2x} \right) \left[\lim_{x \rightarrow 0} \frac{\sin 2x}{\frac{3}{2}(2x)} \right]$$

$$= 1 \cdot \frac{2}{3} \cdot \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} = \frac{2}{3} \cdot 1 = \frac{2}{3}$$

$$70. \lim_{x \rightarrow 0} \frac{\cos x - 1}{\sin x} = \lim_{x \rightarrow 0} \frac{(\cos x - 1)(\cos x + 1)}{\sin x (\cos x + 1)} = \lim_{x \rightarrow 0} \frac{\cos^2 x - 1}{\sin x (\cos x + 1)} = \lim_{x \rightarrow 0} \frac{-\sin^2 x}{\sin x (\cos x + 1)}$$

$$= \lim_{x \rightarrow 0} \frac{-\sin x}{\cos x + 1} = \frac{0}{1+1} = 0$$

$$72. \lim_{x \rightarrow 0} \frac{x}{1 - \cos^2 x} = \lim_{x \rightarrow 0} \frac{x}{\sin^2 x} = \lim_{x \rightarrow 0} \left(\frac{x}{\sin x} \cdot \frac{1}{\sin x} \right) \text{ which does not exist because } \lim_{x \rightarrow 0} \frac{x}{\sin x} = 1,$$

but $\lim_{x \rightarrow 0} \frac{1}{\sin x}$ does not exist.

$$75. \lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 2x} = \lim_{x \rightarrow 0} \left(\frac{\sin 3x}{\sin 2x} \cdot \frac{2x}{2x} \cdot \frac{3}{3} \right) = \lim_{x \rightarrow 0} \left(\frac{\sin 3x}{3x} \cdot \frac{2x}{\sin 2x} \cdot \frac{3}{2} \right)$$

$$= 1 \cdot 1 \cdot \frac{3}{2} = \frac{3}{2}$$

$$= \left(\lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \right) \left(\lim_{x \rightarrow 0} \frac{2x}{\sin 2x} \right) \left(\lim_{x \rightarrow 0} \frac{3}{2} \right)$$

78. Let $t = 2x - \pi$, so that $x = \frac{t + \pi}{2}$. Then

$$\begin{aligned}\lim_{x \rightarrow \pi/2} \frac{\sin(x - \frac{\pi}{2})}{2x - \pi} &= \lim_{(t+\pi)/2 \rightarrow \pi/2} \frac{\sin(\frac{t+\pi}{2} - \frac{\pi}{2})}{t} \\ &= \lim_{t/2 \rightarrow 0} \frac{\sin \frac{t}{2}}{\frac{t}{2} \cdot 2} = \frac{1}{2} \lim_{t/2 \rightarrow 0} \frac{\sin \frac{t}{2}}{\frac{t}{2}} = \frac{1}{2} \cdot 1 = \frac{1}{2}\end{aligned}$$