2.5

2. $g'(x) = \frac{d}{dx}(x + \tan x) = 1 + \sec^2 x$

4. $y' = \frac{d}{dx}(\sqrt{x}\sin x) = x^{1/2}\frac{d}{dx}(\sin x) + \sin x\frac{d}{dx}\left(x^{1/2}\right) = \sqrt{x}\cos x + \frac{\sin x}{2\sqrt{x}}$

8. $f'(t) = \frac{d}{dt}(\sec t\tan t) = \sec t\frac{d}{dt}(\tan t) + \tan t\frac{d}{dt}(\sec t) = (\sec t)\sec^2 t + \tan t(\sec t\tan t) = \sec t\left(\sec^2 t + \tan^2 t\right).$

Alternative answers are $\sec t\left(1 + 2\tan^2 t\right)$ or $\sec t\left(2\sec^2 t - 1\right)$, using the identity $\sec^2 t = 1 + \tan^2 t$.

12. $y' = \frac{d}{d\theta}\left(\frac{\cos\theta}{1-\sin\theta}\right) = \frac{(1-\sin\theta)(-\sin\theta) - \cos\theta(-\cos\theta)}{(1-\sin\theta)^2} = \frac{-\sin\theta + \sin^2\theta + \cos^2\theta}{(1-\sin\theta)^2} = \frac{1-\sin\theta}{(1-\sin\theta)^2}$

$= \frac{1}{1-\sin\theta}$

21. $f'(x) = \frac{d}{dx}\left(x\sin^2 x\right) = x\frac{d}{dx}[(\sin x)(\sin x)] + \sin^2 x\frac{d}{dx}(x) = x\left(\sin x\frac{d}{dx}\sin x + \sin x\frac{d}{dx}\sin x\right) + \sin^2 x$

$= 2x\sin x\cos x + \sin^2 x$ or $x\sin 2x + \sin^2 x$

24. $g'(x) = \frac{d}{dx}(\sec x) = \sec x\tan x \Rightarrow$

$g''(x) = \sec x\frac{d}{dx}(\tan x) + \tan x\frac{d}{dx}(\sec x) = \sec x \cdot \sec^2 x + \tan x \cdot \sec x\tan x = \sec x\left(\sec^2 x + \tan^2 x\right)$

$= \sec x\left(1 + 2\tan^2 x\right)$

26. $h'(t) = \frac{d}{dt}\left[\left(t^2+1\right)\sin t\right] = \left(t^2+1\right)\frac{d}{dt}(\sin t) + \sin t\frac{d}{dt}\left(t^2+1\right) = \left(t^2+1\right)\cos t + 2t\sin t \Rightarrow$

$h''(t) = \frac{d}{dt}\left[\left(t^2+1\right)\cos t + 2t\sin t\right] = \left(t^2+1\right)\frac{d}{dt}(\cos t) + \cos t\frac{d}{dt}\left(t^2+1\right) + 2\left[t\frac{d}{dt}(\sin t) + \sin t\frac{d}{dt}(t)\right]$

$= -\left(t^2+1\right)\sin t + 2t\cos t + 2t\cos t + 2\sin t = 4t\cos t - t^2\sin t + \sin t$

46. $\lim_{h\to 0}\frac{\tan\left(\frac{\pi}{4}+h\right) - 1}{h} = \frac{d}{dx}(\tan x)\Big|_{x=\pi/4}$ (by definition) $= \sec^2 x\Big|_{x=\pi/4} = \left(\sqrt{2}\right)^2 = 2$

2.6

2. $y = \sqrt{x^2 - 4} = g(f(x))$, where $u = f(x) = x^2 - 4$ and $y = g(u) = \sqrt{u} = u^{1/2}$, so
$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \tfrac{1}{2}u^{-1/2} \cdot (2x) = \frac{x}{u^{1/2}} = \frac{x}{\sqrt{x^2 - 4}}.$$

6. $y = \sec\sqrt{x} = g(f(x))$, where $u = f(x) = \sqrt{x}$ and $y = g(u) = \sec x$, so
$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = (\sec u \tan u)\left(\tfrac{1}{2}x^{-1/2}\right) = \frac{\sec\sqrt{x}\tan\sqrt{x}}{2\sqrt{x}}.$$

12. $f(x) = \left(\dfrac{x^2+3}{x}\right)^{-2} \Rightarrow$
$$f'(x) = -2\left(\frac{x^2+3}{x}\right)^{-3}\frac{d}{dx}\left(x + 3x^{-1}\right) = -2\left(\frac{x^2+3}{x}\right)^{-3}\left(1 - \frac{3}{x^2}\right) = -2\left(\frac{x}{x^2+3}\right)^3\left(\frac{x^2-3}{x^2}\right)$$
$$= -\frac{2x\left(x^2-3\right)}{\left(x^2+3\right)^3}$$

20. $f(x) = \left(x^2 + \sqrt{x}\right)^6 = \left(x^2 + x^{1/2}\right)^6 \Rightarrow f'(x) = 6\left(x^2 + x^{1/2}\right)^5\left(2x + \tfrac{1}{2}x^{-1/2}\right) = 6\left(x^2 + \sqrt{x}\right)^5\left(2x + \dfrac{1}{2\sqrt{x}}\right)$

24. $y = \dfrac{(t+1)^3}{\left(t^2+2t\right)^2} = (t+1)^3\left(t^2+2t\right)^{-2} \Rightarrow$
$$\frac{dy}{dt} = (t+1)^3(-2)\left(t^2+2t\right)^{-3}(2t+2) + \left(t^2+2t\right)^{-2}(3)(t+1)^2$$
$$= (t+1)^2\left(t^2+2t\right)^{-3}\left[-4(t+1)^2 + 3\left(t^2+2t\right)\right] = (t+1)^2\left(t^2+2t\right)^{-3}\left(-4t^2 - 8t - 4 + 3t^2 + 6t\right)$$
$$= -\frac{\left(t^2+2t+4\right)(t+1)^2}{t^3(t+2)^3}$$

30. $y = \cos\left(x^3\right) \Rightarrow y' = -\sin\left(x^3\right)\dfrac{d}{dx}\left(x^3\right) = -3x^2\sin\left(x^3\right)$

34. $f(x) = \tan^2 x + \cot\left(x^2\right) \Rightarrow f'(x) = 2\tan x\sec^2 x - \csc^2\left(x^2\right)\dfrac{d}{dx}\left(x^2\right) = 2\tan x\sec^2 x - 2x\csc^2\left(x^2\right)$

38. $g(x) = \tan^2\left(x^2 + x\right) \Rightarrow$

$$g'(x) = 2\tan\left(x^2+x\right)\frac{d}{dx}\tan\left(x^2+x\right) = 2\tan\left(x^2+x\right)\sec^2\left(x^2+x\right)\frac{d}{dx}\left(x^2+x\right)$$
$$= 2(2x+1)\tan\left(x^2+x\right)\sec^2\left(x^2+x\right)$$

48. $y = \sec^3\left(\dfrac{\sqrt{x}}{1+x}\right) \Rightarrow$

$$\frac{dy}{dx} = 3\sec^2\left(\frac{\sqrt{x}}{1+x}\right)\sec\left(\frac{\sqrt{x}}{1+x}\right)\tan\left(\frac{\sqrt{x}}{1+x}\right)\frac{d}{dx}\left(\frac{\sqrt{x}}{1+x}\right)$$
$$= 3\sec^3\left(\frac{\sqrt{x}}{1+x}\right)\tan\left(\frac{\sqrt{x}}{1+x}\right)\cdot\frac{(1+x)\dfrac{1}{2\sqrt{x}} - \sqrt{x}\,(1)}{(1+x)^2} = 3\sec^3\left(\frac{\sqrt{x}}{1+x}\right)\tan\left(\frac{\sqrt{x}}{1+x}\right)\cdot\frac{1+x-2x}{2\sqrt{x}\,(1+x)^2}$$
$$= \frac{3(1-x)}{2\sqrt{x}\,(1+x)^2}\sec^3\left(\frac{\sqrt{x}}{1+x}\right)\tan\left(\frac{\sqrt{x}}{1+x}\right)$$

52. $y = x\tan^2(2x+3) \quad\Rightarrow$

$$\frac{dy}{dx} = x\frac{d}{dx}\tan^2(2x+3) + \tan^2(2x+3)\frac{d}{dx}(x) = x\,[2\tan(2x+3)]\frac{d}{dx}[\tan(2x+3)] + \tan^2(2x+3)$$
$$= 2x\tan(2x+3)\sec^2(2x+3)\frac{d}{dx}(2x+3) + \tan^2(2x+3) = \tan(2x+3)\left[4x\sec^2(2x+3) + \tan(2x+3)\right]$$

66. $F(x) = g(f(x)) \Rightarrow F'(x) = g'(f(x))\,f'(x)$, so $F'(3) = g'(f(3))\,f'(3) = g'(16)\cdot 6 = \frac{1}{8}\cdot 6 = \frac{3}{4}$.

76. $g'(x) = \frac{d}{dx}\left[xf\left(x^2+1\right)\right] = f\left(x^2+1\right) + xf'\left(x^2+1\right)\frac{d}{dx}\left(x^2+1\right) = f\left(x^2+1\right) + 2x^2f'\left(x^2+1\right) \Rightarrow$

$$g''(x) = f'\left(x^2+1\right)\tfrac{d}{dx}\left(x^2+1\right) + 4xf'\left(x^2+1\right) + 2x^2f''\left(x^2+1\right)\tfrac{d}{dx}\left(x^2+1\right)$$
$$= 2xf'\left(x^2+1\right) + 4xf'\left(x^2+1\right) + 4x^3f''\left(x^2+1\right) = 6xf'\left(x^2+1\right) + 4x^3f''\left(x^2+1\right)$$