

## Final Test

## • 考試注意事項

1. 答案紙直行對折，兩直欄書寫作答。
2. 無清楚計算過程，不予計分。

## • 試題

1. (15%) Use the **Second Derivative Test** to find the saddle points of  $f(x, y) = x^3 + y^2 - 2xy + 7x - 8y + 2$ .

2. (15%) Use **Lagrange multipliers** to find the maximum and minimum values of the function  $f(x, y) = x^2 - 2y$  subject to  $x^2 + y^2 = 9$ .

3. (15%) Evaluate  $\int_0^1 \int_y^1 \frac{\sin x}{x} dx dy$ .

4. (15%) Evaluate

$$\iint_R \cos\left(\frac{x-y}{x+y}\right) dA$$

where  $R$  is the trapezoidal region with vertices  $(1, 0)$ ,  $(2, 0)$ ,  $(0, 2)$ , and  $(0, 1)$ .

5. (10%) Let  $f(x, y, z) = xe^{yz}$ . Compute  $f_{xzy}$ .

6. (10%) Let  $w = x^2y + y^2z^3$ , where  $x = r \cos s$ ,  $y = r \sin s$  and  $z = re^s$ . Use the method of the **chain rule** to find the value of  $\partial w / \partial s$  when  $r = 1$  and  $s = 0$ .

7. (10%) Let  $f(x, y) = x^2 - 2xy$ . Find the directional derivative of  $f$  at  $(1, -2)$  in the direction from  $P(-1, 2)$  to  $Q(2, 3)$

8. (10%) Find the equations of the tangent plane to the ellipsoid with equation  $4x^2 + y^2 + 4z^2 = 16$  at the point  $(1, 2, \sqrt{2})$ .