3. $\frac{dw}{dt} = \frac{\partial w}{\partial r}\frac{dr}{dt} + \frac{\partial w}{\partial s}\frac{ds}{dt} = (\cos s + s\cos r)\left(-2e^{-2t}\right) + (-r\sin s + \sin r)\left(3t^2 - 2\right)$
$= -2(\cos s + s\cos r)e^{-2t} + (\sin r - r\sin s)\left(3t^2 - 2\right)$

4. $\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} = \frac{1}{x + y^2}\sec^2 t + \frac{2y}{x + y^2}\sec t\tan t = \frac{\sec t\,(\sec t + 2y\tan t)}{x + y^2}$

9. $\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} = \left(3x^2\right)(2u) + \left(3y^2\right)(2v) = 6\left(x^2u + y^2v\right)$ and
$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} = \left(3x^2\right)(2v) + \left(3y^2\right)(2u) = 6\left(x^2v + y^2u\right)$

11. $\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} = (e^x\cos y)\left(\frac{2u}{u^2 + v^2}\right) + (-e^x\sin y)\left(\frac{1}{2}\frac{\sqrt{uv}}{u}\right) = e^x\left(\frac{2u\cos y}{u^2 + v^2} - \frac{\sqrt{uv}\sin y}{2u}\right)$ and
$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} = (e^x\cos y)\left(\frac{2v}{u^2 + v^2}\right) + (-e^x\sin y)\left(\frac{1}{2}\frac{\sqrt{uv}}{v}\right) = e^x\left(\frac{2v\cos y}{u^2 + v^2} - \frac{\sqrt{uv}\sin y}{2v}\right).$

21. $\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} = \frac{y^2 - x^2}{\left(x^2 + y^2\right)^2}\cdot 2\sec 2t\tan 2t - \frac{2xy}{\left(x^2 + y^2\right)^2}\cdot\sec^2 t$. If $t = 0$, then $x = 1$ and $y = 0$, so
$\left.\frac{du}{dt}\right|_{t=0} = \frac{-1}{1}\cdot 0 - 0 = 0.$

25. $\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial r} = \frac{2xy}{z^2}e^{st} + \frac{x^2}{z^2}ste^{rt} + \left(-\frac{2x^2y}{z^3}\right)ste^{rst}$ and
$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t} = \frac{2xy}{z^2}rse^{st} + \frac{x^2}{z^2}rse^{rt} + \left(-\frac{2x^2y}{z^3}\right)rse^{rst}$. If $r = 1$, $s = 2$, and $t = 0$, then $x = 1$, $y = 2$, and $z = 1$, so $\frac{\partial w}{\partial r} = 4 + 0 + (-4)(0) = 4$ and $\frac{\partial w}{\partial t} = 4(2) + 1(2) - 4(2) = 2$.

29. Here $F(x, y) = x^3 - 2xy + y^3 - 4 = 0$, so $\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{3x^2 - 2y}{-2x + 3y^2} = \frac{3x^2 - 2y}{2x - 3y^2}$.

33. Here $F(x, y, z) = x^2 + xy - x^2z + yz^2 = 0$, so $\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{2x + y - 2xz}{-x^2 + 2yz} = \frac{2x + y - 2xz}{x^2 - 2yz}$ and
$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{x + z^2}{-x^2 + 2yz} = \frac{x + z^2}{x^2 - 2yz}$.

35. Here $F(x, y, z) = xe^y + ye^{xz} + x^2e^{x/y} - 10 = 0$, so
$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{e^y + yze^{xz} + \left(2x + \frac{x^2}{y}\right)e^{x/y}}{xye^{xz}} = -\frac{ye^y + y^2ze^{xz} + x(2y + x)e^{x/y}}{xy^2e^{xz}}$ and
$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{xe^y + e^{xz} + x^2\left(-\frac{x}{y^2}\right)e^{x/y}}{xye^{xz}} = \frac{x^3e^{x/y} - xy^2e^y - y^2e^{xz}}{xy^3e^{xz}}$.

3. Here $\mathbf{u} = \cos\frac{\pi}{2}\,\mathbf{i} + \sin\frac{\pi}{2}\,\mathbf{j} = \mathbf{j}$, so $D_{\mathbf{u}}f(3,0) = \frac{\partial f}{\partial y}(3,0) = (x+1)\,e^{y}\big|_{(3,0)} = 4$.

5. $f_x(x,y) = \frac{\partial}{\partial x}(2x+3xy-3y+4) = 2+3y$ and $f_y(x,y) = 3x-3$, so $\nabla f(2,1) = \left[(2+3y)\,\mathbf{i} + 3(x-1)\,\mathbf{j}\right]_{(2,1)} = 5\mathbf{i}+3\mathbf{j}$.

7. $f_x(x,y) = \frac{\partial}{\partial x}(x\sin y + y\cos x) = \sin y - y\sin x$ and $f_y(x,y) = x\cos y + \cos x$, so $\nabla f\left(\frac{\pi}{4},\frac{\pi}{2}\right) = \left[(\sin y - y\sin x)\,\mathbf{i} + (x\cos y + \cos x)\,\mathbf{j}\right]_{(\pi/4,\pi/2)} = \left[1-\frac{\pi}{2}\left(\frac{\sqrt{2}}{2}\right)\right]\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} = \frac{4-\sqrt{2}\pi}{4}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$.

9. $f_x(x,y,z) = \frac{\partial}{\partial x}\left(xe^{yz}\right) = e^{yz}$, $f_y(x,y,z) = xze^{yz}$, and $f_z(x,y,z) = xye^{yz}$, so $\nabla f(1,0,2) = \left(e^{yz}\mathbf{i} + xze^{yz}\mathbf{j} + xye^{yz}\mathbf{k}\right)\big|_{(1,0,2)} = \mathbf{i}+2\mathbf{j}$.

11. Here $\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{\mathbf{i}-2\mathbf{j}}{\sqrt{1+(-2)^2}} = \frac{\sqrt{5}}{5}\mathbf{i} - \frac{2\sqrt{5}}{5}\mathbf{j}$, $f_x(x,y) = 3x^2-2xy^2+y$, and $f_y(x,y) = -2x^2y+x+2y$, so

$$D_{\mathbf{u}}f(1,-1) = f_x(1,-1)\,u_1 + f_y(1,-1)\,u_2$$
$$= \left[3(1)^2-2(1)(-1)^2+(-1)\right]\left(\frac{\sqrt{5}}{5}\right) + \left[-2(1)^2(-1)+1+2(-1)\right]\left(-\frac{2\sqrt{5}}{5}\right) = -\frac{2\sqrt{5}}{5}$$

15. Here $\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{-\mathbf{i}+3\mathbf{j}}{\sqrt{(-1)^2+3^2}} = -\frac{\sqrt{10}}{10}\mathbf{i} + \frac{3\sqrt{10}}{10}\mathbf{j}$, $f_x(x,y) = -\frac{2y}{(x-y)^2}$, and $f_y(x,y) = \frac{2x}{(x-y)^2}$, so

$$D_{\mathbf{u}}f(2,1) = f_x(2,1)\left(-\frac{\sqrt{10}}{10}\right) + f_y(2,1)\left(\frac{3\sqrt{10}}{10}\right) = -2\left(-\frac{\sqrt{10}}{10}\right) + 4\left(\frac{3\sqrt{10}}{10}\right) = \frac{7\sqrt{10}}{5}.$$

29. Here $\mathbf{u} = \frac{\overrightarrow{PQ}}{\left|\overrightarrow{PQ}\right|} = \frac{\mathbf{i}+3\mathbf{j}}{\sqrt{1^2+3^2}} = \frac{\sqrt{10}}{10}\mathbf{i} + \frac{3\sqrt{10}}{10}\mathbf{j}$, $f_x(x,y) = 3x^2$, and $f_y(x,y) = 3y^2$, so

$$D_{\mathbf{u}}f(1,2) = f_x(1,2)\left(\frac{\sqrt{10}}{10}\right) + f_y(1,2)\left(\frac{3\sqrt{10}}{10}\right) = 3\left(\frac{\sqrt{10}}{10}\right) + 12\left(\frac{3\sqrt{10}}{10}\right) = \frac{39\sqrt{10}}{10}.$$

30. Here $\mathbf{u} = \frac{\overrightarrow{PQ}}{\left|\overrightarrow{PQ}\right|} = \frac{-3\mathbf{i}+2\mathbf{j}}{\sqrt{(-3)^2+2^2}} = -\frac{3\sqrt{13}}{13}\mathbf{i} + \frac{2\sqrt{13}}{13}\mathbf{j}$, $f_x(x,y) = e^{-y}$, and $f_y(x,y) = -xe^{-y}$, so

$$D_{\mathbf{u}}f(2,0) = f_x(2,0)\left(-\frac{3\sqrt{13}}{13}\right) + f_y(2,0)\left(\frac{2\sqrt{13}}{13}\right) = 1\left(-\frac{3\sqrt{13}}{13}\right) - 2\left(\frac{2\sqrt{13}}{13}\right) = -\frac{7\sqrt{13}}{13}.$$

31. Here $\mathbf{u} = \dfrac{\overrightarrow{PQ}}{\left|\overrightarrow{PQ}\right|} = \dfrac{2\mathbf{i} + \frac{\pi}{4}\mathbf{j} - \frac{\pi}{6}\mathbf{k}}{\sqrt{2^2 + \left(\frac{\pi}{4}\right)^2 + \left(\frac{\pi}{6}\right)^2}} = \dfrac{12}{\sqrt{576 + 13\pi^2}}\left(2\mathbf{i} + \frac{\pi}{4}\mathbf{j} - \frac{\pi}{6}\mathbf{k}\right)$, $f_x(x, y, z) = \sin(2y + 3z)$,

$f_y(x, y, z) = 2x\cos(2y + 3z)$, and $f_z(x, y, z) = 3x\cos(2y + 3z)$, so

$$D_{\mathbf{u}} f\left(1, \tfrac{\pi}{4}, -\tfrac{\pi}{12}\right) = f_x\left(1, \tfrac{\pi}{4}, -\tfrac{\pi}{12}\right) \cdot \frac{24}{\sqrt{576 + 13\pi^2}} + f_y\left(1, \tfrac{\pi}{4}, -\tfrac{\pi}{12}\right) \cdot \frac{3\pi}{\sqrt{576 + 13\pi^2}} + f_z\left(1, \tfrac{\pi}{4}, -\tfrac{\pi}{12}\right) \cdot \left(-\frac{2\pi}{\sqrt{576 + 13\pi^2}}\right)$$

$$= \frac{\sqrt{2}}{2}\left(\frac{24}{\sqrt{576 + 13\pi^2}}\right) + \sqrt{2}\left(\frac{3\pi}{\sqrt{576 + 13\pi^2}}\right) + \frac{3\sqrt{2}}{2}\left(-\frac{2\pi}{\sqrt{576 + 13\pi^2}}\right) = \frac{12\sqrt{2}}{\sqrt{13\pi^2 + 576}}$$

32. Here $\mathbf{u} = \dfrac{\overrightarrow{PQ}}{\left|\overrightarrow{PQ}\right|} = \dfrac{\mathbf{i} + \mathbf{j} - 3\mathbf{k}}{\sqrt{1 + 1 + 9}} = \frac{\sqrt{11}}{11}(\mathbf{i} + \mathbf{j} - 3\mathbf{k})$, $f_x(x, y, z) = \dfrac{1}{y + z}$, $f_y(x, y, z) = \dfrac{z - x}{(y + z)^2}$, and

$f_z(x, y, z) = -\dfrac{x + y}{(y + z)^2}$, so

$$D_{\mathbf{u}} f(2, 1, 1) = f_x(2, 1, 1)\left(\tfrac{\sqrt{11}}{11}\right) + f_y(2, 1, 1)\left(\tfrac{\sqrt{11}}{11}\right) + f_z(2, 1, 1)\left(-\tfrac{3\sqrt{11}}{11}\right) = \tfrac{1}{2}\left(\tfrac{\sqrt{11}}{11}\right) - \tfrac{1}{4}\left(\tfrac{\sqrt{11}}{11}\right) - \tfrac{3}{4}\left(-\tfrac{3\sqrt{11}}{11}\right)$$

$$= \tfrac{5\sqrt{11}}{22}$$