

3.6

7. $f(x) = x^3 - 6x^2 + 9x + 2$

(1) The domain of f is $(-\infty, \infty)$. (2) The y -intercept is 2. The x -intercepts are not easily found, so we will not use this information.

(3) There is no symmetry. (4) $\lim_{x \rightarrow -\infty} f(x) = -\infty$ and

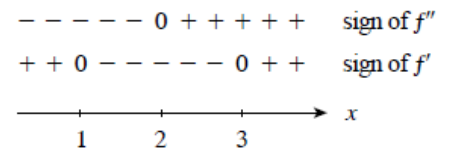
$\lim_{x \rightarrow \infty} f(x) = \infty$. (5) f is a polynomial function, so there is no asymptote.

(6) $f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x - 3)(x - 1) = 0$

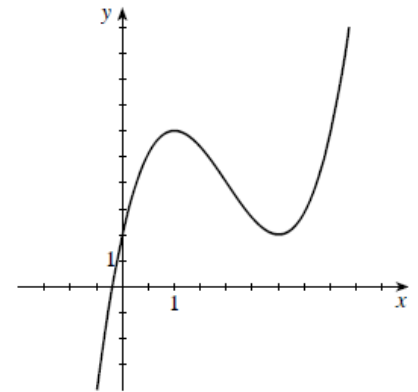
$\Leftrightarrow x = 1$ or 3 , the critical numbers of f . From the sign diagram for f' , we see that f is increasing on $(-\infty, 1)$ and $(3, \infty)$ and decreasing on $(1, 3)$.

(7) From (6), we know that 1 and 3 are critical numbers of f . f' changes sign from positive to negative as we move across $x = 1$, so f has a relative maximum of $f(1) = 6$, and similarly, $f(3) = 2$ is a relative minimum.

(8) $f''(x) = 6x - 12 = 6(x - 2) = 0 \Leftrightarrow x = 2$. The sign diagram for f'' shows that f is concave downward on $(-\infty, 2)$ and concave upward on $(2, \infty)$. (9) From (8), we see that f'' changes sign as we move across $x = 2$. Since $f(2) = 4$, f has an inflection point at $(2, 4)$.



(10)



11. $g(x) = x^4 + 2x^3 - 2$

(1) The domain of g is $(-\infty, \infty)$. (2) The y -intercept is -2 .

(3) There is no symmetry. (4) $\lim_{x \rightarrow -\infty} g(x) = \lim_{x \rightarrow \infty} g(x) = \infty$.

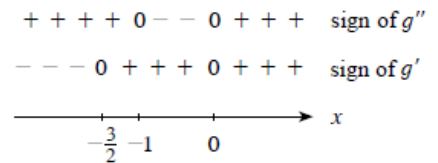
(5) There is no asymptote. (6) $g'(x) = 4x^3 + 6x^2 = 2x^2(2x + 3) = 0$

$\Leftrightarrow x = 0$ or $-\frac{3}{2}$, the critical numbers of f . From the sign diagram for g' ,

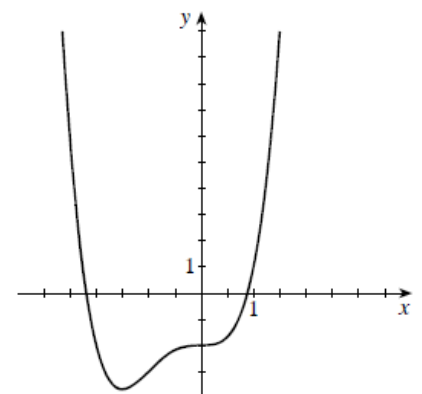
we see that g is decreasing on $(-\infty, -\frac{3}{2})$ and increasing on $(-\frac{3}{2}, \infty)$.

(7) g has a relative minimum at $(-\frac{3}{2}, -\frac{59}{16})$.

(8) $g''(x) = 12x^2 + 12x = 12x(x + 1) = 0 \Leftrightarrow x = 0$ or -1 . From the sign diagram of g'' , we see that g is concave upward on $(-\infty, -1)$ and $(0, \infty)$ and concave downward on $(-1, 0)$. (9) g has inflection points at $(-1, -3)$ and $(0, -2)$.



(10)



19. $h(x) = \frac{x}{x^2 - 9}$

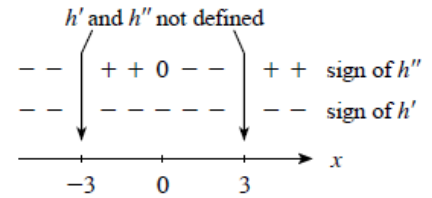
(1) The domain of h is $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$. (2) The x - and y -intercepts are 0. (3) $h(-x) = \frac{-x}{(-x)^2 - 9} = -\frac{x}{x^2 - 9} = -h(x)$, so the graph of h is symmetric with respect to the origin.

(4) $\lim_{x \rightarrow -\infty} h(x) = 0$ and $\lim_{x \rightarrow \infty} h(x) = 0$. (5) From (4), we see that $y = 0$ is a horizontal asymptote. Since $\lim_{x \rightarrow -3^-} h(x) = \lim_{x \rightarrow 3^-} h(x) = -\infty$ and $\lim_{x \rightarrow -3^+} h(x) = \lim_{x \rightarrow 3^+} h(x) = \infty$, $x = \pm 3$ are vertical asymptotes.

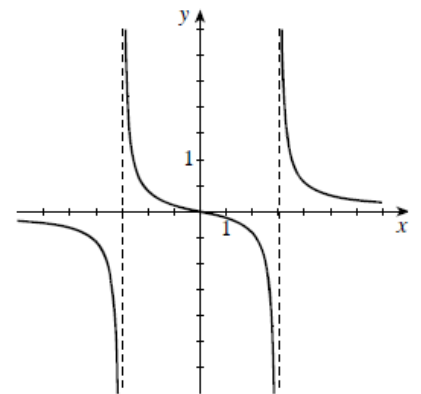
(6) $h'(x) = \frac{(x^2 - 9) - x(2x)}{(x^2 - 9)^2} = -\frac{x^2 + 9}{(x^2 - 9)^2}$. From the sign diagram

for h' we see that h is decreasing on its domain. (7) f has no relative extremum.

(8) $h''(x) = -\frac{(x^2 - 9)^2(2x) - (x^2 + 9)2(x^2 - 9)(2x)}{(x^2 - 9)^4} = \frac{2x(x^2 + 27)}{(x^2 - 9)^3}$. From the sign diagram of h'' , we see that h is concave downward on $(-\infty, -3)$ and $(0, 3)$ and concave upward on $(-3, 0)$ and $(3, \infty)$. (9) h has an inflection point at $(0, 0)$. Neither of ± 3 is in the domain of h .



(10)



23. $h(x) = \frac{1}{x^2 - x - 2}$

(1) Note that the denominator $x^2 - x - 2 = (x - 2)(x + 1) = 0 \Leftrightarrow x = -1$ or $x = 2$, so the domain of h is $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$.

(2) Setting $x = 0$ gives $-\frac{1}{2}$ as the y -intercept. (3) There is no symmetry.

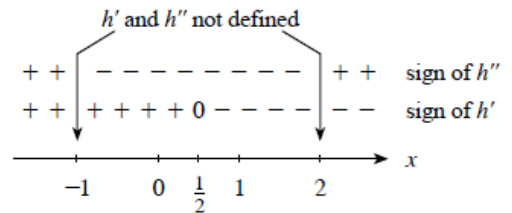
(4) $\lim_{x \rightarrow -\infty} h(x) = \lim_{x \rightarrow \infty} h(x) = 0$ (5) The results of (4) show that $y = 0$ is a horizontal asymptote. Furthermore, the denominator is 0 at $x = -1$ or 2 , where the numerator is not equal to 0. Therefore, $x = -1$ and $x = 2$ are vertical asymptotes.

(6) $f'(x) = \frac{d}{dx} (x^2 - x - 2)^{-1} = -(x^2 - x - 2)^{-2} (2x - 1) = \frac{1 - 2x}{(x^2 - x - 2)^2}$

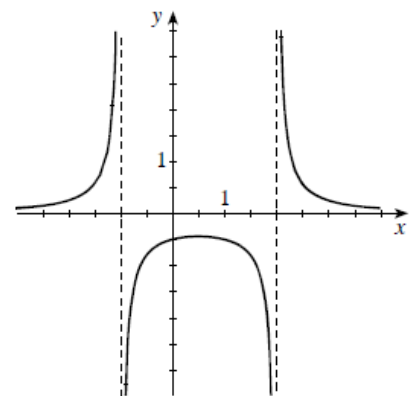
Setting $f'(x) = 0$ gives $x = \frac{1}{2}$. The sign diagram of h' shows that h is increasing on $(-\infty, -1)$ and $(-1, \frac{1}{2})$ and decreasing on $(\frac{1}{2}, 2)$ and $(2, \infty)$. (7) The results of (5) show that $(\frac{1}{2}, -\frac{4}{9})$ is a relative maximum.

(8) $h''(x) = \frac{(x^2 - x - 2)^2(-2) - (1 - 2x)2(x^2 - x - 2)(2x - 1)}{(x^2 - x - 2)^4} = \frac{2(x^2 - x - 2)[-(x^2 - x - 2) + (2x - 1)^2]}{(x^2 - x - 2)^4} = \frac{6(x^2 - x + 1)}{(x^2 - x - 2)^3}$

$h''(x)$ has no zero and is discontinuous at $x = -1$ and $x = 2$. The sign diagram of h'' shows that the graph of h is concave upward on $(-\infty, -1)$ and $(2, \infty)$ and concave downward on $(-1, 2)$. (9) There is no inflection point.



(10)



Chapter 3 Review

3. $h(x) = x^3 - 6x^2$ is continuous on $[2, 5]$. $h'(x) = 3x^2 - 12x = 3x(x - 4) = 0 \Leftrightarrow x = 0$ or $x = 4$, so 4 is the only critical number of h in $(2, 5)$. $h(2) = -16$, $h(4) = -32$, and $h(5) = -25$, so the absolute maximum value is -16 attained at 2, and the absolute minimum value is -32 attained at 4.

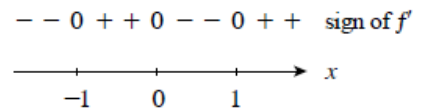
5. $f(x) = 4x - \frac{1}{x^2}$ is continuous on $[1, 3]$. $f'(x) = 4 + \frac{2}{x^3} = \frac{4x^3 + 2}{x^3} = \frac{2(2x^3 + 1)}{x^3} = 0 \Rightarrow x = -\frac{1}{\sqrt[3]{2}}$, so f has no critical number in $(1, 3)$. $f(1) = 3$ and $f(3) = \frac{107}{9}$, so the absolute maximum value is $\frac{107}{9}$ attained at 3, and the absolute minimum value is 3 attained at 1.

9. $f(x) = \cos x - \sin x$ is continuous on $[0, 2\pi]$. $f'(x) = -\sin x - \cos x = 0 \Leftrightarrow \tan x = -1 \Rightarrow x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$ in $(0, 2\pi)$. $f(0) = 1$, $f\left(\frac{3\pi}{4}\right) = -\sqrt{2}$, $f\left(\frac{7\pi}{4}\right) = \sqrt{2}$, and $f(2\pi) = 1$, so the absolute maximum value is $\sqrt{2}$ attained at $\frac{7\pi}{4}$, and the absolute minimum value is $-\sqrt{2}$ attained at $\frac{3\pi}{4}$.

23. a. $f(x) = x^4 - 2x^2 \Rightarrow$

$$f'(x) = 4x^3 - 4x = 4x(x^2 - 1) = 4x(x + 1)(x - 1). \text{ The sign}$$

diagram of f' shows that f is decreasing on $(-\infty, -1)$ and $(0, 1)$ and increasing on $(-1, 0)$ and $(1, \infty)$.

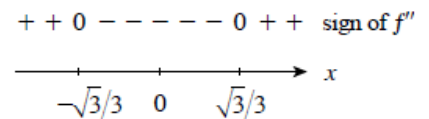


b. The results of part a and the First Derivative Test show that $(-1, -1)$ and $(1, -1)$ are relative minima and $(0, 0)$ is a relative maximum.

c. $f''(x) = 12x^2 - 4 = 4(3x^2 - 1) = 0 \Rightarrow x = \pm\frac{\sqrt{3}}{3}$. The sign diagram

shows that f is concave upward on $(-\infty, -\frac{\sqrt{3}}{3}) \cup (\frac{\sqrt{3}}{3}, \infty)$ and

concave downward on $(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$.

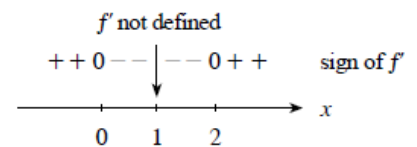


d. The results of part c show that $(-\frac{\sqrt{3}}{3}, -\frac{5}{9})$ and $(\frac{\sqrt{3}}{3}, -\frac{5}{9})$ are inflection points.

25. a. $f(x) = \frac{x^2}{x-1} \Rightarrow$

$$f'(x) = \frac{(x-1)(2x) - x^2(1)}{(x-1)^2} = \frac{x^2 - 2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}. \text{ The sign}$$

diagram of f' shows that f is increasing on $(-\infty, 0)$ and $(2, \infty)$ and decreasing on $(0, 1)$ and $(1, 2)$.



b. The results of part a show that $(0, 0)$ is a relative maximum and $(2, 4)$ is a relative minimum.

c. $f''(x) = \frac{(x-1)^2(2x-2) - x(x-2)2(x-1)}{(x-1)^4} = \frac{2(x-1)[(x-1)^2 - x(x-2)]}{(x-1)^4} = \frac{2}{(x-1)^3}$. Since $f''(x) < 0$

if $x < 1$ and $f''(x) > 0$ if $x > 1$, we see that f is concave downward on $(-\infty, 1)$ and concave upward on $(1, \infty)$.

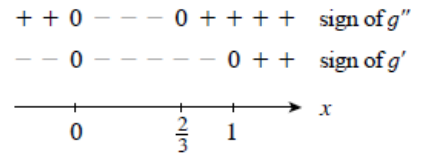
d. Since $x = 1$ is not in the domain of f , there is no inflection point.

43. $\lim_{x \rightarrow -\infty} \frac{1}{2x+3} = \lim_{x \rightarrow \infty} \frac{1}{2x+3} = 0$ and so $y = 0$ is a horizontal asymptote. Since the denominator is equal to zero at

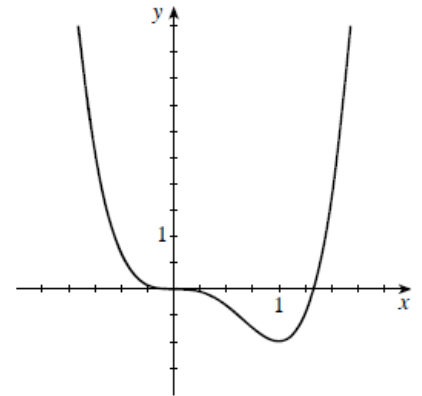
$x = -\frac{3}{2}$ but the numerator is not, we see that $x = -\frac{3}{2}$ is a vertical asymptote.

47. $g(x) = 3x^4 - 4x^3$

- (1) The domain of g is $(-\infty, \infty)$. (2) $g(x) = x^3(3x - 4) = 0 \Leftrightarrow x = 0$ or $\frac{4}{3}$, the x -intercepts. The y -intercept is 0. (3) There is no symmetry. (4) $\lim_{x \rightarrow -\infty} g(x) = \lim_{x \rightarrow \infty} g(x) = \infty$ (5) There is no asymptote. (6) $g'(x) = 12x^3 - 12x^2 = 12x^2(x - 1) = 0 \Leftrightarrow x = 0$ or $x = 1$, the critical numbers of f . From the sign diagram for g' , we see that g is decreasing on $(-\infty, 1)$ and increasing on $(1, \infty)$. (7) g has a relative minimum at $(1, -1)$. (8) $g''(x) = 36x^2 - 24x = 12x(3x - 2) = 0 \Leftrightarrow x = 0$ or $\frac{2}{3}$. From the sign diagram of g'' , we see that g is concave upward on $(-\infty, 0)$ and $(\frac{2}{3}, \infty)$ and concave downward on $(0, \frac{2}{3})$. (9) g has inflection points at $(0, 0)$ and $(\frac{2}{3}, -\frac{16}{27})$.

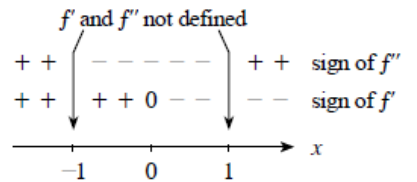


(10)

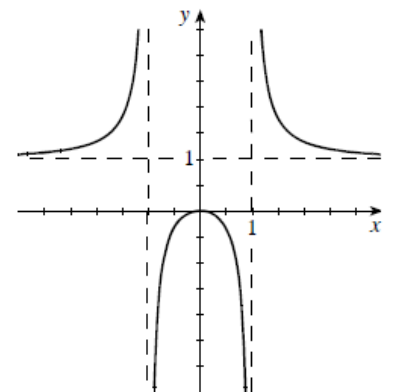


51. $f(x) = \frac{x^2}{x^2 - 1}$

- (1) The domain of f is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$. (2) The x - and y -intercepts are 0. (3) $f(-x) = \frac{(-x)^2}{(-x)^2 - 1} = \frac{x^2}{x^2 - 1} = f(x)$, so there is symmetry with respect to the y -axis. (4) $\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow \infty} f(x) = 1$ (5) $x = \pm 1$ are vertical asymptotes. From (4), we see that $y = 1$ is a horizontal asymptote. (6) $f'(x) = \frac{(x^2 - 1)(2x) - x^2(2x)}{(x^2 - 1)^2} = -\frac{2x}{(x^2 - 1)^2}$. From the sign diagram for f' , we see that f is increasing on $(-\infty, -1)$ and $(-1, 0)$ and decreasing on $(0, 1)$ and $(1, \infty)$. (7) f has a relative maximum at $(0, 0)$.



(10)



(8) $f''(x) = -2 \frac{d}{dx} \frac{x}{(x^2 - 1)^2} = -2 \left[\frac{(x^2 - 1)^2 - x(2)(x^2 - 1)(2x)}{(x^2 - 1)^4} \right] = \frac{2(3x^2 + 1)}{(x^2 - 1)^3}$. From the sign diagram of f'' ,

we see that f is concave upward on $(-\infty, -1)$ and $(1, \infty)$ and concave downward on $(-1, 1)$ (9) f has no inflection point because ± 1 lie outside the domain of f .

55. $h(x) = 2 \sin x - \sin 2x, 0 \leq x \leq 2\pi$

(1) The domain of h is $[0, 2\pi]$. (2) $h(0) = 0$, so the y -intercept is 0.

There are x -intercepts at $0, \pi$, and 2π . (3) There is no symmetry.

(4) Not applicable (5) There is no asymptote.

(6) $h'(x) = 2 \cos x - 2 \cos 2x = 2 \cos x - 2(2 \cos^2 x - 1) = 0 \Leftrightarrow$

$2 \cos^2 x - \cos x - 1 = (2 \cos x + 1)(\cos x - 1) = 0 \Rightarrow x = \frac{2\pi}{3}, \frac{4\pi}{3},$ or

π . From the sign diagram for f' , we see that f is increasing on $(0, \frac{2\pi}{3})$

and $(\frac{4\pi}{3}, 2\pi)$ and decreasing on $(\frac{2\pi}{3}, \frac{4\pi}{3})$. (7) h has a relative

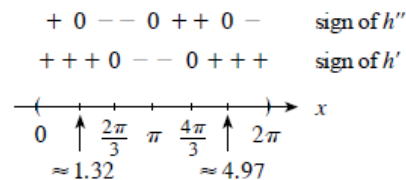
maximum at $(\frac{2\pi}{3}, \frac{3\sqrt{3}}{2})$ and a relative minimum at $(\frac{4\pi}{3}, -\frac{3\sqrt{3}}{2})$.

(8) $h''(x) = -2 \sin x + 4 \sin 2x = -2 \sin x + 8 \sin x \cos x = 0 \Leftrightarrow$

$\sin x(1 - 4 \cos x) = 0 \Leftrightarrow x = \cos^{-1} \frac{1}{4} \approx 1.32, \pi,$ or

$2\pi - \cos^{-1} \frac{1}{4} \approx 4.97$. From the sign diagram of h'' , we see that h is concave upward on approximately $(0, 1.32)$ and

$(\pi, 4.97)$ and concave downward on approximately $(1.32, \pi)$ and $(4.97, 2\pi)$. (9) h has inflection points at approximately $(1.32, 1.46)$ and $(4.97, -1.44)$ and exactly $(\pi, 0)$.



(10)

