1.1 An Intuitive Introduction to Limits

2. **a.** $\lim_{x\to 2^-} f(x) = 2$

b. $\lim_{x\to 2^+} f(x) = -1$

c. $\lim_{x\to 2} f(x)$ does not exist.

4. **a.** $\lim_{x\to 3^-} f(x) = 2$

b. $\lim_{x\to 3^+} f(x) = 2$

c. $\lim_{x\to 3} f(x) = 2$

8. **a.** True.

b. True. $\lim_{x\to 0^-} f(x) = 3 = \lim_{x\to 0^+} f(x)$

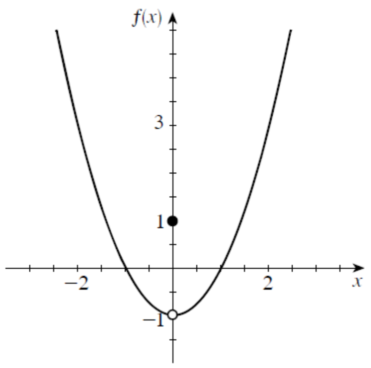
c. True.

d. True.

e. False. Since $\lim_{x\to 3^-} f(x) = 3$ and $\lim_{x\to 3^+} f(x) = 3$, $\lim_{x\to 3} f(x) = 3$.

f. True.

20.

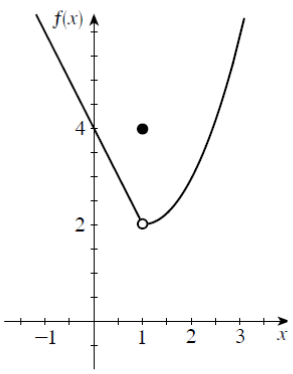


a. $\lim_{x\to 0^-} f(x) = -1$

b. $\lim_{x\to 0^+} f(x) = -1$

c. $\lim_{x\to 0} f(x) = -1$

22.



a. $\lim_{x\to 1^-} f(x) = 2$

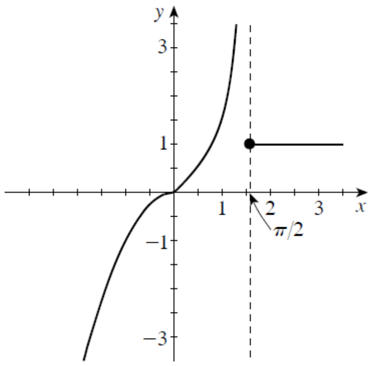
b. $\lim_{x\to 1^+} f(x) = 2$

c. $\lim_{x\to 1} f(x) = 2$

24. $\lim_{x\to 3^+} [\![x]\!] = 3$

26. $\lim_{x\to-1} [\![x]\!]$ does not exist.

32. a.



b. The limit of $f(x)$ exists for all values of x in $\left(-\infty, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \infty\right)$.

c. The left-hand limit of $f(x)$ exists for all values of x in $\left(-\infty, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \infty\right)$.

d. The right-hand limit of $f(x)$ exists for all values of x in $(-\infty, \infty)$

1.2 Techniques for Finding Limits

12. $\lim_{x\to-2} (x+3)^2 \sqrt{4x^2-8} = 1^2\sqrt{4(-2)^2-8} = 2\sqrt{2}$

22. $\lim_{x\to\pi/4} \dfrac{\tan^2 x}{1+\cos x} = \dfrac{\left(\tan\frac{\pi}{4}\right)^2}{1+\cos\frac{\pi}{4}} = \dfrac{1^2}{1+\frac{\sqrt{2}}{2}} = \dfrac{2}{2+\sqrt{2}} = 2-\sqrt{2}$

28. $\lim_{x\to a} \dfrac{\sqrt[3]{f(x)g(x)}}{\sqrt{f(x)g(x)+1}} = \dfrac{\sqrt[3]{\lim_{x\to a} f(x)\cdot \lim_{x\to a} g(x)}}{\sqrt{\lim_{x\to a} f(x)\cdot \lim_{x\to a} g(x)+1}} = \dfrac{\sqrt[3]{2\cdot 4}}{\sqrt{2\cdot 4+1}} = \dfrac{2}{3}$

42. $\lim_{x\to 5} \dfrac{5-x}{x^2-25} = \lim_{x\to 5} \dfrac{(-1)(x-5)}{(x+5)(x-5)} = (-1)\lim_{x\to 5} \dfrac{1}{x+5} = -\dfrac{1}{10}$

58. $$\lim_{h\to 0} \frac{\sqrt{a+h}-\sqrt{a}}{h} = \lim_{h\to 0} \frac{\sqrt{a+h}-\sqrt{a}}{h} \cdot \frac{\sqrt{a+h}+\sqrt{a}}{\sqrt{a+h}+\sqrt{a}} = \lim_{h\to 0} \frac{a+h-a}{h\left(\sqrt{a+h}+\sqrt{a}\right)}$$
$$= \lim_{h\to 0} \frac{1}{\sqrt{a+h}+\sqrt{a}} = \frac{1}{2\sqrt{a}} = \frac{\sqrt{a}}{2a}$$

60. $\lim_{h\to 0} \dfrac{(2+h)^{-1}-2^{-1}}{h} = \lim_{h\to 0} \dfrac{\frac{1}{2+h}-\frac{1}{2}}{h} = \lim_{h\to 0} \dfrac{2-(2+h)}{(2+h)\,2h} = \lim_{h\to 0} \dfrac{-1}{2(2+h)} = -\dfrac{1}{4}$

66. $\lim_{x\to 0} \dfrac{\sin 2x}{x} = \lim_{x\to 0} \left(\dfrac{\sin 2x}{2x}\cdot\dfrac{2}{1}\right) = 2\cdot \lim_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 2\cdot 1 = 2$. We have made the substitution $\theta = 2x$ at the third step.

68. $\lim_{x\to 0}\frac{\tan 2x}{3x} = \lim_{x\to 0}\left(\frac{\sin 2x}{\cos 2x}\cdot\frac{1}{3x}\right) = \left(\lim_{x\to 0}\frac{1}{\cos 2x}\right)\left[\lim_{x\to 0}\frac{\sin 2x}{\frac{3}{2}(2x)}\right] = 1\cdot\frac{2}{3}\cdot\lim_{x\to 0}\frac{\sin 2x}{2x} = \frac{2}{3}\cdot 1 = \frac{2}{3}$

70. $\lim_{x\to 0}\frac{\cos x - 1}{\sin x} = \lim_{x\to 0}\frac{(\cos x - 1)(\cos x + 1)}{\sin x\,(\cos x + 1)} = \lim_{x\to 0}\frac{\cos^2 x - 1}{\sin x\,(\cos x + 1)} = \lim_{x\to 0}\frac{-\sin^2 x}{\sin x\,(\cos x + 1)}$

$= \lim_{x\to 0}\frac{-\sin x}{\cos x + 1} = \frac{0}{1+1} = 0$

72. $\lim_{x\to 0}\frac{x}{1-\cos^2 x} = \lim_{x\to 0}\frac{x}{\sin^2 x} = \lim_{x\to 0}\left(\frac{x}{\sin x}\cdot\frac{1}{\sin x}\right)$ which does not exist because $\lim_{x\to 0}\frac{x}{\sin x} = 1$, but $\lim_{x\to 0}\frac{1}{\sin x}$ does not exist.

75. $\lim_{x\to 0}\frac{\sin 3x}{\sin 2x} = \lim_{x\to 0}\left(\frac{\sin 3x}{\sin 2x}\cdot\frac{2x}{2x}\cdot\frac{3}{3}\right) = \lim_{x\to 0}\left(\frac{\sin 3x}{3x}\cdot\frac{2x}{\sin 2x}\cdot\frac{3}{2}\right) = \left(\lim_{x\to 0}\frac{\sin 3x}{3x}\right)\left(\lim_{x\to 0}\frac{2x}{\sin 2x}\right)\left(\lim_{x\to 0}\frac{3}{2}\right)$

$= 1\cdot 1\cdot\frac{3}{2} = \frac{3}{2}$

78. Let $t = 2x - \pi$, so that $x = \frac{t+\pi}{2}$. Then

$$\lim_{x\to\pi/2}\frac{\sin\left(x-\frac{\pi}{2}\right)}{2x-\pi} = \lim_{(t+\pi)/2\to\pi/2}\frac{\sin\left(\frac{t+\pi}{2}-\frac{\pi}{2}\right)}{t} = \lim_{t/2\to 0}\frac{\sin\frac{t}{2}}{\frac{t}{2}\cdot 2} = \frac{1}{2}\lim_{t/2\to 0}\frac{\sin\frac{t}{2}}{\frac{t}{2}} = \frac{1}{2}\cdot 1 = \frac{1}{2}$$