# Calculus

2014/05/13

考試注意事項：

1. 答案紙直行對折，兩直欄書寫作答。
2. 無清楚計算過程，不予計分。

試題：

1. (15%) Show that $\lim_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$ does not exist.

2. (15%) Let $f(x,y,z) = xe^{yz}$. Compute $f_{xy}$, $f_{yz}$, and $f_{zx}$.

3. (15%) Let $w = x^2y + y^2z^3$, where $x = r\cos s$, $y = r\sin s$, and $z = re^s$. Find the value of $\partial w/\partial s$ when $r = 1$ and $s = 0$.

4. (15%) Find the directional derivative of $f(x,y) = e^x \cos 2y$ at the point $(0, \frac{\pi}{4})$ in the direction $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$.

5. (10%) Find an equation of the tangent line to the curve

$$x = \sec t \quad y = \tan t \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

at the point where $t = \pi/4$.

6. (10%) Use the cross product of two vectors to find parametric equations for the line of intersection of the planes defined by $3x - y + 2z = 1$ and $2x + 3y - z = 4$.

7. (10%) Evaluate $\displaystyle\int_0^1 \mathbf{r}(t)\,dt$ if $\mathbf{r}(t) = t^2\mathbf{i} + \frac{1}{1+t}\mathbf{j} + e^{-t}\mathbf{k}$.

8. (10%) Find the length of the arc of the curve $C$ given by the vector function $\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j} + t\mathbf{k}$, where $0 \le t \le 2\pi$.