2.2

12. $g'(x) = \frac{d}{dx}\left(-\frac{1}{3}x^2 + \sqrt{2}x\right) = -\frac{2}{3}x + \sqrt{2}$

22. $f'(x) = \frac{d}{dx}\left(5x^{4/3} - \frac{2}{3}x^{3/2} + x^2 - 3x + 1\right) = \frac{20}{3}x^{1/3} - x^{1/2} + 2x - 3$

24. $f'(x) = \frac{d}{dx}\left[-\frac{1}{3}\left(x^{-3} - x^6\right)\right] = -\frac{1}{3}\left(-3x^{-4} - 6x^5\right) = x^{-4} + 2x^5 = 2x^5 + \frac{1}{x^4}$

26. $f'(x) = \frac{d}{dx}\left(5x^{-3} - 2x^{-2} - x^{-1} + 200\right) = -15x^{-4} + 4x^{-3} + x^{-2}$

$$= -\frac{15}{x^4} + \frac{4}{x^3} + \frac{1}{x^2}$$

32. $f'(u) = \frac{d}{du}\left(u^{-1/2} - 3u^{-1/3}\right) = -\frac{1}{2}u^{-3/2} + u^{-4/3} = -\frac{1}{2u^{3/2}} + \frac{1}{u^{4/3}}$

33. $f'(x) = \frac{d}{dx}\left(2x^3 - 4x\right) = 6x^2 - 4$

a. $f'(-2) = 6(-2)^2 - 4 = 24 - 4 = 20$

b. $f'(0) = 6(0) - 4 = -4$

c. $f'(2) = 6(2)^2 - 4 = 20$

40. $g'(x) = x^2 - x - 1 = -1 \Rightarrow x^2 - x = x(x-1) = 0 \Rightarrow x = 0$ or 1.

$g(0) = 1$ and $g(1) = \frac{1}{3} - \frac{1}{2} - 1 + 1 = -\frac{1}{6}$, so the points are $(0, 1)$ and $\left(1, -\frac{1}{6}\right)$.

42. $F'(s) = \frac{d}{ds}\left(2 + \frac{1}{s}\right) = -\frac{1}{s^2} = -\frac{1}{9} \Rightarrow s^2 = 9 \Rightarrow s = \pm 3$.

$F(3) = \frac{2(3)+1}{3} = \frac{7}{3}$ and $F(-3) = \frac{2(-3)+1}{-3} = \frac{5}{3}$, so

the points are $\left(3, \frac{7}{3}\right)$ and $\left(-3, \frac{5}{3}\right)$.

50. The line $y = 2x$ has slope 2. Also $y = x^2 + c \Rightarrow \dfrac{dy}{dx} = 2x$,

so $2x = 2 \Rightarrow x = 1$. Therefore $y = 2$. Substituting into the

second equation gives $2 = 1 + c$, so $c = 1$.

54. $\displaystyle\lim_{h\to 0} \frac{x^5 - 1}{x - 1} = \lim_{h\to 0} \frac{(1+h)^5 - 1}{h} = f'(1)$, where $f(x) = x^5$.

$f'(x) = 5x^4$, so $f'(1) = 5$.

58. We use the definition of the derivative to write

$$\lim_{h\to 0} \frac{\dfrac{1}{x+h} + \sqrt{x+h} - \dfrac{1}{x} - \sqrt{x}}{h} = \lim_{h\to 0} \frac{\dfrac{1}{x+h} - \dfrac{1}{x}}{h} + \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \frac{d}{dx}\left(\frac{1}{x}\right) + \frac{d}{dx}\left(\sqrt{x}\right) = -\frac{1}{x^2} + \frac{1}{2\sqrt{x}}.$$

72. In order for f to be continuous at a, $\displaystyle\lim_{x\to a^-} f(x) = \lim_{x\to a^-} x^2 = a^2$

must be equal to $\displaystyle\lim_{x\to a^+} f(x) = \lim_{x\to a^+} (Ax + B) = Aa + B$,

that is, $aA + B = a^2$. In order for f to be differentiable at a, we must have

$$\lim_{h\to 0^-} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0^+} \frac{f(a+h) - f(a)}{h} \text{ or } 2a = A.$$

Therefore, $A = 2a$ and $B = a^2 - 2a^2 = -a^2$.

2.3

4. $f'(x) = (3x+1)\dfrac{d}{dx}\left(x^2-2\right) + \left(x^2-2\right)\dfrac{d}{dx}(3x+1)$

$= (3x+1)(2x) + \left(x^2-2\right)(3) = 6x^2 + 2x + 3x^2 - 6$

$= 9x^2 + 2x - 6$

6. $f'(w) = \left(w^3 - w^2 + w - 1\right)\dfrac{d}{dw}\left(w^3+2\right) + \left(w^3+2\right)\dfrac{d}{dw}\left(w^3 - w^2 + w - 1\right)$

$= \left(w^3 - w^2 + w - 1\right)\left(3w^2\right) + \left(w^3+2\right)\left(3w^2 - 2w + 1\right)$

$= 3w^5 - 3w^4 + 3w^3 - 3w^2 + 3w^5 - 2w^4 + w^3 + 6w^2 - 4w + 2$

$= 6w^5 - 5w^4 + 4w^3 + 3w^2 - 4w + 2$

10. $P'(t) = \dfrac{(3-2t)\dfrac{d}{dt}(2-t) - (2-t)\dfrac{d}{dt}(3-2t)}{(3-2t)^2}$

$= \dfrac{(3-2t)(-1) - (2-t)(-2)}{(3-2t)^2} = \dfrac{-3+2t+4-2t}{(3-2t)^2} = \dfrac{1}{(3-2t)^2}$

12. $f'(s) = \dfrac{(s+1)\dfrac{d}{ds}\left(s^2-4\right) - \left(s^2-4\right)\dfrac{d}{ds}(s+1)}{(s+1)^2}$

$= \dfrac{(s+1)(2s) - \left(s^2-4\right)(1)}{(s+1)^2} = \dfrac{2s^2 + 2s - s^2 + 4}{(s+1)^2}$

$= \dfrac{s^2 + 2s + 4}{(s+1)^2}$

19. $$f'(x)=\frac{\left(x^2+1\right)\frac{d}{dx}\left(2x^{1/2}\right)-\left(2x^{1/2}\right)\frac{d}{dx}\left(x^2+1\right)}{\left(x^2+1\right)^2}=\frac{\left(x^2+1\right)\left(x^{-1/2}\right)-\left(2x^{1/2}\right)(2x)}{\left(x^2+1\right)^2}$$

$$=\frac{x^{3/2}+x^{-1/2}-4x^{3/2}}{\left(x^2+1\right)^2}=\frac{-3x^{3/2}+x^{-1/2}}{\left(x^2+1\right)^2}=\frac{x^{-1/2}\left(-3x^2+1\right)}{\left(x^2+1\right)^2}=\frac{-3x^2+1}{\sqrt{x}\left(x^2+1\right)^2}$$

24. $$f'(r)=\frac{d}{dr}\left[\frac{(2r+1)(r-3)}{3r+1}\right]=\frac{d}{dr}\left(\frac{2r^2-5r-3}{3r+1}\right)$$

$$=\frac{(3r+1)\frac{d}{dr}\left(2r^2-5r-3\right)-\left(2r^2-5r-3\right)\frac{d}{dr}(3r+1)}{(3r+1)^2}$$

$$=\frac{(3r+1)(4r-5)-\left(2r^2-5r-3\right)(3)}{(3r+1)^2}$$

$$=\frac{\left(12r^2-15r+4r-5\right)-\left(6r^2-15r-9\right)}{(3r+1)^2}=\frac{6r^2+4r+4}{(3r+1)^2}$$

$$=\frac{2\left(3r^2+2r+2\right)}{(3r+1)^2}$$

38. $$f(x)=\frac{x}{x^2+1}\Rightarrow f'(x)=\frac{\left(x^2+1\right)(1)-x(2x)}{\left(x^2+1\right)^2}=\frac{1-x^2}{\left(x^2+1\right)^2}.$$

At a point where the tangent line is horizontal, we have $f'(x)=0\Rightarrow 1-x^2=0$,

giving $x=\pm1$. Therefore, the required points are $\left(-1,-\frac{1}{2}\right)$ and $\left(1,\frac{1}{2}\right)$.

48. $$h'(x)=\left(x^2+1\right)g'(x)+g(x)(2x)$$

$$\Rightarrow h'(1)=2g'(1)+g(1)(2)=(2)(3)+(-2)(2)=2$$

52. Take $f(x) = (x+1)^2$. Then $f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x-1} = \lim_{x \to 1} \frac{(x+1)^2 - 4}{x-1}$.

But $f'(x) = \frac{d}{dx}(x+1)^2 = \frac{d}{dx}\left(x^2 + 2x + 1\right) = 2x + 2$.

Therefore, $\lim_{x \to 1} \frac{(x+1)^2 - 4}{x-1} = f'(1) = (2x+2)|_{x=1} = 4$.

66. a. The rate of change of the pond's oxygen level at any time t is given by

$$\begin{aligned}
f'(t) &= 100\frac{d}{dt}\left(\frac{t^2+10t+100}{t^2+20t+100}\right) \\
&= 100\left[\frac{\left(t^2+20t+100\right)\frac{d}{dt}\left(t^2+10t+100\right) - \left(t^2+10t+100\right)\frac{d}{dt}\left(t^2+20t+100\right)}{\left(t^2+20t+100\right)^2}\right] \\
&= 100\left[\frac{\left(t^2+20t+100\right)(2t+10) - \left(t^2+10t+100\right)(2t+20)}{\left(t^2+20t+100\right)^2}\right] = 1000\left[\frac{t^2-100}{\left(t^2+20t+100\right)^2}\right] \\
&= \frac{1000(t-10)}{(t+10)^3}
\end{aligned}$$

b. $f'(1) = \frac{1000(1-10)}{(1+10)^3} \approx -6.76$, $f'(10) = \frac{1000(10-10)}{(1+10)^3} = 0$, and $f'(20) = \frac{1000(20-10)}{(1+10)^3} \approx 0.37$.

c. One day after the organic waste has been dumped into the pond the oxygen content was dropping at the rate of approximately 6.8% per day. Ten days after the waste was dumped, the oxygen content was neither increasing or decreasing. After 20 days, the oxygen content was increasing at the rate of approximately 0.37% per day.

76. True. If $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ is a polynomial of degree n, then

$$\begin{aligned}
P'(x) &= na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + \cdots + 3a_3x^2 + 2a_2x + a_1 \\
P''(x) &= n(n-1)a_n x^{n-2} + (n-1)(n-2)a_{n-1}x^{n-3} + \cdots + 6a_3x + 2a_2 \\
&\vdots \\
P^{(n-1)}(x) &= n(n-1)(n-2)\cdots(2)x \\
P^{(n)}(x) &= n(n-1)(n-2)\cdots(2)(1) \\
P^{(n+1)}(x) &= 0
\end{aligned}$$