7.1

6. $\int x^3 \ln x\, dx$. Let $u = \ln x$ and $dv = x^3\, dx$. Then $du = dx/x$ and $v = \int x^3\, dx = \frac{1}{4}x^4$, so

$$\int x^3 \ln x\, dx = uv - \int v\, du = \tfrac{1}{4}x^4 \ln x - \int \tfrac{1}{4}x^3\, dx = \tfrac{1}{4}x^4 \ln x - \tfrac{1}{16}x^4 + C = \tfrac{1}{16}x^4 (4\ln x - 1) + C.$$

11. $\int \tan^{-1} x\, dx$. Let $u = \tan^{-1} x$ and $dv = dx$. Then $du = \dfrac{dx}{1+x^2}$ and $v = x$, so

$$\int \tan^{-1} x\, dx = uv - \int v\, du = x\tan^{-1} x - \int \frac{x\, dx}{1+x^2} = x\tan^{-1} x - \tfrac{1}{2}\ln\left(1+x^2\right) + C.$$

29. $\int e^{-x} \ln(e^x+1)\, dx$. Let $u = \ln(e^x+1)$ and $dv = e^{-x}\, dx$, so $du = \dfrac{e^x}{e^x+1}$ and $v = -e^{-x}$. Then

$$\int e^{-x}\ln(e^x+1)\, dx = uv - \int v\, du = -e^{-x}\ln(e^x+1) + \int \frac{dx}{e^x+1} = -e^{-x}\ln(e^x+1) + \int \frac{e^{-x}}{1+e^{-x}}\, dx$$
$$= -e^{-x}\ln(e^x+1) - \ln(1+e^{-x}) + C$$

33. $\int_1^e x^2 \ln x\, dx$. Let $u = \ln x$ and $dv = x^2\, dx$, so $du = dx/x$ and $v = \frac{1}{3}x^3$. Then

$$\int_1^e x^2 \ln x\, dx = \tfrac{1}{3}x^3 \ln x\Big|_1^e - \tfrac{1}{3}\int_1^e x^2\, dx = \tfrac{1}{3}e^3 - \tfrac{1}{9}\left[x^3\right]_1^e = \tfrac{1}{3}e^3 - \tfrac{1}{9}\left(e^3 - 1\right) = \tfrac{1}{9}\left(2e^3+1\right).$$

37. $\int_{\sqrt{e}}^e x^{-2}\ln x\, dx$. Let $u = \ln x$ and $dv = x^{-2}\, dx$, so $du = dx/x$ and $v = -1/x$. Then

$$\int_{\sqrt{e}}^e x^{-2}\ln x\, dx = -\frac{\ln x}{x}\Big|_{\sqrt{e}}^e + \int_{\sqrt{e}}^e x^{-2}\, dx = -\frac{1}{e} + \frac{\ln\sqrt{e}}{\sqrt{e}} - \left[\frac{1}{x}\right]_{\sqrt{e}}^e = -\frac{1}{e} + \frac{1}{2\sqrt{e}} - \frac{1}{e} + \frac{1}{\sqrt{e}} = \frac{3}{2\sqrt{e}} - \frac{2}{e} = \frac{3\sqrt{e}-4}{2e}.$$

7. 2

6. $\int \cos^3 2x\, dx = \int \cos^2 2x \cdot \cos 2x\, dx = \int \left(1 - \sin^2 2x\right) \cos 2x\, dx$. Let $u = \sin 2x$, so $du = 2\cos 2x\, dx$. Then $\int \cos^3 2x\, dx = \frac{1}{2} \int \left(1 - u^2\right) du = \frac{1}{2}\left(u - \frac{1}{3}u^3\right) + C = \frac{1}{2}\left(\sin 2x - \frac{1}{3}\sin^3 2x\right) + C$.

10.
$$\begin{aligned}
\int \sin^2 2x \cos^4 2x\, dx &= \int \tfrac{1}{2}(1 - \cos 4x)\left[\tfrac{1}{2}(1 + \cos 4x)\right]^2 dx = \tfrac{1}{8}\int (1 - \cos 4x)\left(1 + 2\cos 4x + \cos^2 4x\right) dx \\
&= \tfrac{1}{8}\int \left(1 + 2\cos 4x + \cos^2 4x - \cos 4x - 2\cos^2 4x - \cos^3 4x\right) dx \\
&= \tfrac{1}{8}\int \left(1 + \cos 4x - \cos^2 4x - \cos^3 4x\right) dx = \tfrac{1}{8}\int \left[\sin^2 4x + (\cos 4x)\left(1 - \cos^2 4x\right)\right] dx \\
&= \tfrac{1}{8}\int \left[\tfrac{1}{2}(1 - \cos 8x) + \sin^2 4x \cos 4x\right] dx = \tfrac{1}{8}\int \left(\tfrac{1}{2} - \tfrac{1}{2}\cos 8x + \sin^2 4x \cos 4x\right) dx \\
&= \tfrac{1}{16}\left(x - \tfrac{1}{8}\sin 8x + \tfrac{1}{6}\sin^3 4x\right) + C
\end{aligned}$$

19. $\int_0^{\pi/4} \tan^2 x\, dx = \int_0^{\pi/4} \left(\sec^2 x - 1\right) dx = (\tan x - x)\big|_0^{\pi/4} = 1 - \frac{\pi}{4} = \frac{4 - \pi}{4}$

22.
$$\begin{aligned}
\int \tan^5 x \sec^3 x\, dx &= \int \tan^4 x \sec^2 x \sec x \tan x\, dx = \int \left(\sec^2 x - 1\right)^2 \sec^2 x \sec x \tan x\, dx \\
&= \int \left(\sec^6 x - 2\sec^4 x + \sec^2 x\right) \sec x \tan x\, dx = \tfrac{1}{7}\sec^7 x - \tfrac{2}{5}\sec^5 x + \tfrac{1}{3}\sec^3 x + C
\end{aligned}$$

44. $\displaystyle\int_0^{\pi/2} \frac{\sin t}{1 + \cos t}\, dt$. Let $u = 1 + \cos t$, so $du = -\sin t\, dt$, $t = 0 \Rightarrow u = 2$, and $t = \frac{\pi}{2} \Rightarrow u = 1$. Then $\displaystyle\int_0^{\pi/2} \frac{\sin t}{1 + \cos t}\, dt = -\int_2^1 \frac{du}{u} = -\ln u\big|_2^1 = -\ln 1 + \ln 2 = \ln 2.$