2.6 The Chain Rule

23. $g(x) = \left(\frac{2x^2-1}{2x+5}\right)^{1/3} \Rightarrow$

$$g'(x) = \frac{1}{3}\left(\frac{2x^2-1}{2x+5}\right)^{-2/3} \cdot \frac{(2x+5)(4x)-\left(2x^2-1\right)(2)}{(2x+5)^2} = \frac{1}{3}\left(\frac{2x^2-1}{2x+5}\right)^{-2/3} \cdot \frac{4x^2+20x+2}{(2x+5)^2}$$
$$= \frac{2\left(2x^2+10x+1\right)}{3\,(2x^2-1)^{2/3}(2x+5)^{4/3}}$$

31. $f(x) = \sin 2x + \tan\sqrt{x} \Rightarrow f'(x) = (\cos 2x)\,2 + \left(\sec^2\sqrt{x}\right)\left(\frac{1}{2}x^{-1/2}\right) = 2\cos 2x + \frac{\sec^2\sqrt{x}}{2\sqrt{x}}$

67. No. Let $F(x) = f(f(x))$. Then $F'(x) = f'(f(x))\,f'(x)$. If we let $f(x) = x^2$, then $F(x) = f(f(x)) = f\left(x^2\right) = x^4$, so $F'(x) = 4x^3$, but $[f'(x)]^2 = (2x)^2 = 4x^2$.

74. $F'(x) = \frac{d}{dx}\left\{f(x^a) + [f(x)]^a\right\} = \frac{d}{dx}f(x^a) + \frac{d}{dx}[f(x)]^a = f'(x^a)\frac{d}{dx}(x^a) + a[f(x)]^{a-1}\frac{d}{dx}f(x)$
$= ax^{a-1}f'(x^a) + af'(x)[f(x)]^{a-1}$

2.7 Implicit Differentiation

12. $\left(2x^2+3y^2\right)^{5/2} = x \Rightarrow \left(2x^2+3y^2\right) = x^{2/5} \Rightarrow 4x + 6yy' = \frac{2}{5}x^{-3/5} \Rightarrow 6yy' = \frac{2}{5x^{3/5}} - 4x = \frac{2\left(1-10x^{8/5}\right)}{5x^{3/5}} \Rightarrow$
$y' = \frac{1-10x^{8/5}}{15x^{3/5}y}$

22. $x^2y + y^3 = 2 \Rightarrow 2xy + x^2y' + 3y^2y' = 0 \Rightarrow y' = -\frac{2xy}{x^2+3y^2}$, so $y'|_{(-1,1)} = -\frac{2(-1)(1)}{1+3} = \frac{1}{2}$. An equation of the tangent line is $y - 1 = \frac{1}{2}(x+1)$ or $y = \frac{1}{2}x + \frac{3}{2}$.

26. $x^{2/3} + y^{2/3} = 5 \Rightarrow \frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}y' = 0$. At $(1,8)$, $\frac{2}{3} + \frac{2}{3}\cdot 8^{-1/3}y' = 0 \Rightarrow y' = -2$.

32. $\tan y - xy = 0 \Rightarrow \left(\sec^2 y\right)y' - y - xy' = 0 \Rightarrow y' = \frac{y}{\sec^2 y - x}$. Differentiating both sides of the next-to-last expression yields $\left(2\sec^2 y\tan y\right)(y')^2 + \left(\sec^2 y\right)y'' - y' - y' - xy'' = 0 \Rightarrow$