Calculus

考試注意事項：

   1. 答案紙直行對折，兩直攔書寫作答。

   2. 無清楚計算過程，不予計分。

試題：

1. (10%,5%) Let
$$f(x) = \begin{cases} x\sin\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

   Show that

   (a) f is continuous at $x = 0$;

   (b) f is not differentiable at $x = 0$.

2. (15%) Suppose that f and g are functions that are differentiable at $x = 1$ and that $f(1) = 2$, $f'(1) = -1$, $g(1) = -2$, and $g'(-1) = 3$. Find $h'(1)$ where h is given by
$$h(x) = \frac{f(x)g(x)}{f(x) - g(x)}.$$

3. (15%) Find $\dfrac{dy}{dx}$ if $y = \tan^3(3x^2 + 1)$.

4. (15%) Find $\dfrac{dy}{dx}$ if $x + y^2 = \cos xy$.

5. (5%,5%)

   (a) Find $\displaystyle\lim_{x \to 3} \frac{4x^2 - 3x + 1}{2x - 4}$.

   (b) Find $\displaystyle\lim_{x \to 5} \frac{5 - x}{x^2 - 25}$.

6. (10%) Let
$$f(x) = \begin{cases} \frac{\sin 2x}{x} & \text{if } x \neq 0 \\ c & \text{if } x = 0 \end{cases}$$

   Find the value of c that will make f continuous on $(-\infty, \infty)$.

7. (10%) Find an equation of the tangent line to the graph of $y = x\sin x$ at the point where $x = \pi/2$

8. (10%) Find the linearization of a suitable function, then use it to approximate the number $\sqrt{63.8}$.