

5. Along  $y = 0$ ,  $\lim_{(x,y) \rightarrow (0,0)} \frac{2xy}{\sqrt{x^4+y^4}} = \lim_{x \rightarrow 0} \frac{0}{\sqrt{x^4}} = \lim_{x \rightarrow 0} 0 = 0$ . Along  $y = x$ ,  
 $\lim_{(x,y) \rightarrow (0,0)} \frac{2xy}{\sqrt{x^4+y^4}} = \lim_{x \rightarrow 0} \frac{2x^2}{\sqrt{x^4+x^4}} = \lim_{x \rightarrow 0} \frac{2x^2}{\sqrt{2x^4}} = \lim_{x \rightarrow 0} \frac{2x^2}{\sqrt{2}x^2} = \sqrt{2}$ . Because these two limits are not equal, the given limit does not exist.
9. Along the  $x$ -axis,  $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xy+yz+xz}{x^2+y^2+z^2} = \lim_{x \rightarrow 0} \frac{0}{x^2} = 0$ . Let  $C$  denote the curve with parametric equations  $x = t, y = t, z = t$ . Then along  $C$ ,  $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xy+yz+xz}{x^2+y^2+z^2} = \lim_{t \rightarrow 0} \frac{3t^2}{3t^2} = \lim_{t \rightarrow 0} 1 = 1$ . Because these two limits are not equal, the given limit does not exist.
15.  $\lim_{(x,y) \rightarrow (1,2)} \frac{2x^2 - 3y^3 + 4}{3 - xy} = \frac{2(1)^2 - 3(2)^3 + 4}{3 - (1)(2)} = -18$
19.  $\lim_{(x,y) \rightarrow (0^+, 0^+)} \frac{e^{\sqrt{x+y}}}{x+y-1} = \frac{e^0}{-1} = -1$

## 13.3 Partial Derivatives

17.  $g_u(u, v) = \frac{\partial}{\partial u} \left( \frac{uv}{u^2 + v^3} \right) = \frac{(u^2 + v^3)(v) - (uv)(2u)}{(u^2 + v^3)^2} = \frac{v(v^3 - u^2)}{(u^2 + v^3)^2}$  and  
 $g_v(u, v) = \frac{\partial}{\partial v} \left( \frac{uv}{u^2 + v^3} \right) = \frac{(u^2 + v^3)(u) - (uv)(3v^2)}{(u^2 + v^3)^2} = \frac{u(u^2 - 2v^3)}{(u^2 + v^3)^2}$ .
26.  $f_u(u, v, w) = \frac{\partial}{\partial u} (ue^v - ve^u + we^u) = e^v - ve^u + we^u$ ,  $f_v(u, v, w) = \frac{\partial}{\partial v} (ue^v - ve^u + we^u) = ue^v - e^u$ , and  
 $f_w(u, v, w) = \frac{\partial}{\partial w} (ue^v - ve^u + we^u) = e^u$ .
28.  $\frac{\partial u}{\partial x} = \frac{\partial}{\partial x} x \sin \frac{y}{x+z} = \sin \frac{y}{x+z} + \left( x \cos \frac{y}{x+z} \right) \frac{\partial}{\partial x} [y(x+z)^{-1}] = \sin \frac{y}{x+z} + \left( x \cos \frac{y}{x+z} \right) [-y(x+z)^{-2}]$   
 $= \sin \frac{y}{x+z} - \frac{xy}{(x+z)^2} \cos \frac{y}{x+z}$ ,  
 $\frac{\partial u}{\partial y} = \frac{\partial}{\partial y} x \sin \frac{y}{x+z} = \left( x \cos \frac{y}{x+z} \right) \frac{\partial}{\partial y} \left( \frac{y}{x+z} \right) = \frac{x}{x+z} \cos \frac{y}{x+z}$ , and  
 $\frac{\partial u}{\partial z} = \frac{\partial}{\partial z} x \sin \frac{y}{x+z} = x \cos \frac{y}{x+z} \frac{\partial}{\partial z} [y(x+z)^{-1}] = \left( x \cos \frac{y}{x+z} \right) [-y(x+z)^{-2}]$   
 $= -\frac{xy}{(x+z)^2} \cos \frac{y}{x+z}$
68.  $\frac{\partial u}{\partial x} = \frac{\partial}{\partial x} \left( 20x^2 \cos \frac{y}{x} \right) = 40x \cos \frac{y}{x} + 20x^2 \left( -\sin \frac{y}{x} \right) \left( -\frac{y}{x^2} \right) = 20 \left( 2x \cos \frac{y}{x} + y \sin \frac{y}{x} \right)$   
and  $\frac{\partial u}{\partial y} = \frac{\partial}{\partial y} 20x^2 \cos \frac{y}{x} = 20x^2 \left( -\sin \frac{y}{x} \right) \left( \frac{1}{x} \right) = -20x \sin \frac{y}{x}$ . Therefore,  
 $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 40x^2 \cos \frac{y}{x} + 20xy \sin \frac{y}{x} - 20xy \sin \frac{y}{x} = 40x^2 \cos \frac{y}{x} = 2u$ , as was to be shown.