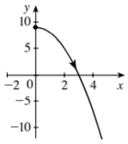
10.2 Plane Curves and Parametric Equations

3. a. $\left.\begin{array}{r} x=\sqrt{t} \\ y=9-t \end{array}\right\} \Rightarrow y=9-x^2, x \geq 0$

b.



The orientation is found by observing that as t increases, so does x.

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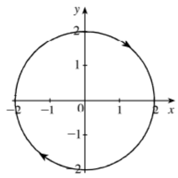
9. a. $\left.\begin{array}{r} x=2\sin\theta \\ y=2\cos\theta \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \sin\theta=\frac{1}{2}x \\ \cos\theta=\frac{1}{2}y \end{array}\right\} \Rightarrow$

$\left(\frac{1}{2}x\right)^2+\left(\frac{1}{2}y\right)^2=\sin^2\theta+\cos^2\theta=1$, so $x^2+y^2=4$.

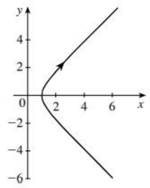
b.



Observe that as θ increases from 0 to 2π, the curve C is traced once in a clockwise direction starting from the point $(0, 2)$.

17. a. $\left.\begin{array}{r} x-\sec\theta \\ y=\tan\theta \end{array}\right\} \Rightarrow \left\{\begin{array}{l} x^2-\sec^2\theta \\ y^2=\tan^2\theta \end{array}\right\}$ From the identity $\sec^2\theta=1+\tan^2\theta$, we obtain $x^2-y^2=1, x\geq 1$.

b.



37. a. $x=x_1+(x_2-x_1)t$ and $y=y_1+(y_2-y_1)t$. From the first equation, we find $t=\dfrac{x-x_1}{x_2-x_1}$. Substituting this expression into the second equation gives $y=y_1+(y_2-y_1)\left(\dfrac{x-x_1}{x_2-x_1}\right)=y_1+\dfrac{y_2-y_1}{x_2-x_1}x-\dfrac{x_1(y_2-y_1)}{x_2-x_1}$, which is a linear equation in x and y. Since (x_1, y_1) and (x_2, y_2) both satisfy the equation, we see that $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ both lie on the line.

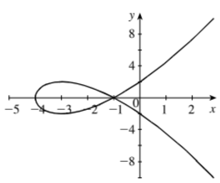
b. If $t=0$, $x=x_1$ and $y=y_1$, so (x_1, y_1) is on the line. If $t=1$, then $x=x_2$ and $y=y_2$, so (x_2, y_2) is on the line. As t increases from $t=0$ to $t=1$, the line segment joining $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is traced out.

10.3 The Calculus of Parametric Equations

4. $x=e^{2t}$, $y=\ln t \Rightarrow \frac{dx}{dt}=2e^{2t}$ and $\frac{dy}{dt}=1/t$. The slope of the tangent line at $t=1$ is $\left.\dfrac{dy}{dx}\right|_{t=1}=\left.\dfrac{dy/dt}{dx/dt}\right|_{t=1}=\left.\dfrac{1/t}{2e^{2t}}\right|_{t=1}=\dfrac{1}{2e^2}$.

6. $x=2(\theta-\sin\theta)$, $y=2(1-\cos\theta) \Rightarrow \frac{dx}{d\theta}=2(1-\cos\theta)$ and $\frac{dy}{d\theta}=2\sin\theta$. The slope of the tangent line at $\theta=\frac{\pi}{6}$ is $\left.\dfrac{dy}{dx}\right|_{\theta=\pi/6}=\left.\dfrac{dy/d\theta}{dx/d\theta}\right|_{\theta=\pi/6}=\left.\dfrac{2\sin\theta}{2(1-\cos\theta)}\right|_{\theta=\pi/6}=\dfrac{1}{2\left(1-\frac{\sqrt{3}}{2}\right)}=\dfrac{1}{2-\sqrt{3}}=2+\sqrt{3}$.

13. $x=t^2-4$, $y=t^3-3t \Rightarrow \frac{dx}{dt}=2t$ and $\frac{dy}{dt}=3t^2-3$. To find the point(s) where the tangent line is horizontal, set $\frac{dy}{dt}=0 \Rightarrow 3t^2-3=3(t+1)(t-1)=0 \Rightarrow t=\pm 1$. Since $\frac{dx}{dt}\neq 0$ at either of these t-values, the required points are $(x(-1), y(-1))=(-3, 2)$ and $(x(1), y(1))=(-3, -2)$. To find the point(s) where the tangent line is vertical, set $\frac{dx}{dt}=0 \Rightarrow 2t=0 \Rightarrow t=0$. Since $\frac{dy}{dt}\neq 0$ at this value of t, we see that the required point is $(x(0), y(0))=(-4, 0)$.



19. $x=\sqrt{t}$, $y=\dfrac{1}{t} \Rightarrow \dfrac{dx}{dt}=\dfrac{1}{2\sqrt{t}}$ and $\dfrac{dy}{dt}=-\dfrac{1}{t^2}$, so $\dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}=\dfrac{-1/t^2}{1/(2t^{1/2})}=-2t^{-3/2}=-\dfrac{2}{t^{3/2}}$ and

$$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}=\frac{\frac{d}{dt}\left(-2t^{-3/2}\right)}{1/(2t^{1/2})}=3t^{-5/2}\cdot 2t^{1/2}=\frac{6}{t^2}.$$