Calculus II

2017/05/02

Second Midterm

- 考試注意事項
  1. 答案紙直行對折，兩直欄書寫作答。
  2. 無清楚計算過程，不予計分。
- 試題

0. (15%) Using alternating series test to show the series $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^n}$ converges.

1. (15%) Find the radius of convergence and the interval of convergence of $\sum_{n=1}^{\infty} \frac{x^n}{n}$.

2. (15%) Find the Taylor series for $f(x) = \ln x$ at 1, and determine its interval of convergence.

3. (15%) Find $\frac{d^2y}{dx^2}$ if $x = t^2 - 4$ and $y = t^3 - 3t$.

4. (10%) Write $\mathbf{b} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ as the sum of a vector parallel to $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ and a vector perpendicular to $\mathbf{a}$.

5. (10%) Find the volume of the parallelepiped determined by the vectors $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$, and $\mathbf{c} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

6. (10%) Find an equation of the plane containing the points $P(3, -1, 1)$, $Q(1, 4, 2)$ and $R(0, 1, 4)$.

7. (10%) Find the antiderivative of $\mathbf{r}' = \cos t\mathbf{i} + e^{-t}\mathbf{j} + \sqrt{t}\mathbf{k}$ satisfying the initial condition $\mathbf{r}(0) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.