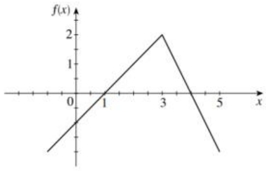
1 Limits and Continuity

1.1 Concept Questions

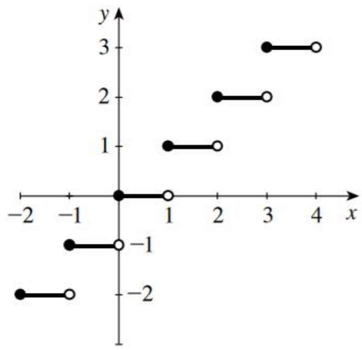
17.



a. $\lim_{x\to 3^-} f(x) = 2$

b. $\lim_{x\to 3^+} f(x) = 2$

c. $\lim_{x\to 3} f(x) = 2$



26. $\lim_{x\to -1} [\![x]\!]$ does not exist.

34. From the given figure, for $n = 1, 2, 3, \ldots$, we have $\lim_{t\to[(2n-1)k]^-} f(t) = k$ and $\lim_{t\to[(2n-1)k]^+} f(t) = 0$, so $\lim_{t\to[(2n-1)k]} f(t)$ does not exist. Similarly, $\lim_{t\to[(2n)k]^-} f(t) = 0$ and $\lim_{t\to[(2n)k]^+} f(t) = k$, so $\lim_{t\to[(2n)k]} f(t)$ does not exist. Therefore, $\lim_{t\to nk} f(t)$ does not exist for $n = 1, 2, 3, \ldots$.

1.2 Techniques for Finding Limits

25. $\lim_{x\to a} \dfrac{f(x)}{\sqrt{g(x)}} = \dfrac{\lim_{x\to a} f(x)}{\sqrt{\lim_{x\to a} g(x)}} = \dfrac{2}{\sqrt{4}} = 1$

30. $\lim_{x\to -2} \dfrac{xf(x)}{1+x^2} = \dfrac{\left(\lim_{x\to -2} x\right)\left[\lim_{x\to -2} f(x)\right]}{\lim_{x\to -2} 1 + \left(\lim_{x\to -2} x\right)^2} = \dfrac{(-2)\,2}{1+(-2)^2} = -\dfrac{4}{5}$

45. $\lim_{x\to 1} \dfrac{x^2+2x-3}{x^2-1} = \lim_{x\to 1} \dfrac{(x-1)(x+3)}{(x+1)(x-1)} = \lim_{x\to 1} \dfrac{x+3}{x+1} = 2$

57. $\lim_{x\to 0} \dfrac{\sqrt{x+3}-\sqrt{3}}{x} = \lim_{x\to 0} \dfrac{\sqrt{x+3}-\sqrt{3}}{x} \cdot \dfrac{\sqrt{x+3}+\sqrt{3}}{\sqrt{x+3}+\sqrt{3}} = \lim_{x\to 0} \dfrac{x+3-3}{x\left(\sqrt{x+3}+\sqrt{3}\right)}$

$= \lim_{x\to 0} \dfrac{1}{\sqrt{x+3}+\sqrt{3}} = \dfrac{1}{2\sqrt{3}} = \dfrac{\sqrt{3}}{6}$

90. $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} |x| = 1$ and $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} [\![x]\!] = 1$. Since $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = 1$, we conclude that $\lim_{x\to 1} f(x)$ exists and has a value of 1.