Not homework. Just supplement.

14.2 Iterated Integrals ET 13.2

3. $\int_0^2 \int_1^4 y\sqrt{x}\,dy\,dx = \int_0^2 \left[\int_1^4 yx^{1/2}\,dy\right] dx = \int_0^2 \left[\frac{1}{2}y^2x^{1/2}\right]_{y=1}^{y=4} dx = \int_0^2 \frac{15}{2}x^{1/2}\,dx = 5x^{3/2}\Big|_0^2 = 10\sqrt{2}$

5. $\int_0^{\pi} \int_0^{\pi} \cos(x+y)\,dy\,dx = \int_0^{\pi} \left[\int_0^{\pi} \cos(x+y)\,dy\right] dx = \int_0^{\pi} \left[\sin(x+y)\right]_{y=0}^{y=\pi} dx = \int_0^{\pi} \left[\sin(x+\pi) - \sin x\right] dx$

$= -2\int_0^{\pi} \sin x\,dx = 2\cos x\big|_0^{\pi} = -4$

11. $\int_{-1}^1 \int_x^{2x} e^{x+y}\,dy\,dx = \int_{-1}^1 \left[\int_x^{2x} e^{x+y}\,dy\right] dx = \int_{-1}^1 \left[e^{x+y}\right]_{y=x}^{y=2x} dx = \int_{-1}^1 \left(e^{3x} - e^{2x}\right) dx = \left(\frac{1}{3}e^{3x} - \frac{1}{2}e^{2x}\right)\Big|_{-1}^1$

$= \left(\frac{1}{3}e^3 - \frac{1}{2}e^2\right) - \left(\frac{1}{3}e^{-3} - \frac{1}{2}e^{-2}\right) = \frac{2e^6 - 3e^5 + 3e - 2}{6e^3}$

12. $\int_0^{\pi} \int_{e^{-2x}}^{e^{\cos x}} \frac{\ln y}{y}\,dy\,dx = \int_0^{\pi} \left[\int_{e^{-2x}}^{e^{\cos x}} \frac{\ln y}{y}\,dy\right] dx = \int_0^{\pi} \left[\frac{1}{2}(\ln y)^2\right]_{y=e^{-2x}}^{y=e^{\cos x}} dx = \frac{1}{2}\int_0^{\pi} \left[\left(\ln e^{\cos x}\right)^2 - \left(\ln e^{-2x}\right)^2\right] dx$

$= \frac{1}{2}\int_0^{\pi} \left(\cos^2 x - 4x^2\right) dx = \frac{1}{2}\int_0^{\pi} \left[\frac{1}{2} + \frac{1}{2}\cos 2x - 4x^2\right] dx$

$= \frac{1}{2}\left(\frac{1}{2}x + \frac{1}{4}\sin 2x - \frac{4}{3}x^3\right)\Big|_0^{\pi} = \frac{1}{12}\pi\left(3 - 8\pi^2\right)$

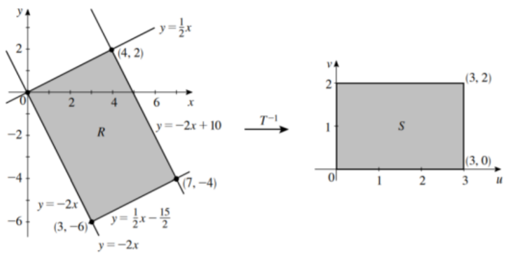
14.8 Change of Variables in Multiple Integrals ET 13.8

13. To find T^{-1}, we solve the system of equations of $T: x = u + 2v,\ y = v - 2u$ for u and v, obtaining $T^{-1}: u = \frac{1}{5}(x - 2y),\ v = \frac{1}{5}(2x + y)$. Using this transformation, we obtain the region $S = T^{-1}(R)$. Next, we find

$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = 5$. Thus,

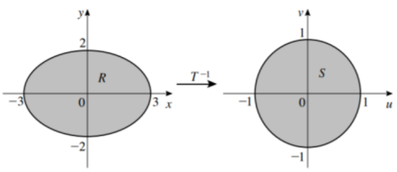
$\iint_R (x+y)\,dA = \iint_S \left[(u+2v) + (v-2u)\right] \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv = 5\int_0^2 \int_0^3 (3v - u)\,du\,dv$

$= 5\int_0^2 \left[3uv - \frac{1}{2}u^2\right]_{u=0}^{u=3} dv = 5\int_0^2 \left(9v - \frac{9}{2}\right) dv = 5\left(\frac{9}{2}v^2 - \frac{9}{2}v\right)\Big|_0^2 = 45$



15. Here $T: x = 3u,\ y = 2v$. Then $4x^2 + 9y^2 = 36$ $\Rightarrow 4(3u)^2 + 9(2v)^2 = 36 \Leftrightarrow u^2 + v^2 = 1$, the circle with radius 1 centered at the origin of the uv-plane. Next, we find

$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ 0 & 2 \end{vmatrix} = 6$, so



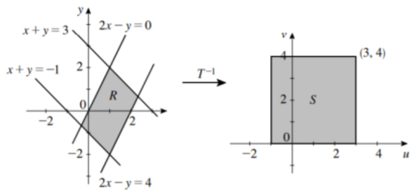
$\iint_R 2xy\,dA = 2\iint_S (3u)(2v) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv = 12 \cdot 6\int_0^{2\pi} \int_0^1 (r\cos\theta)(r\sin\theta)\,r\,dr\,d\theta = 72\int_0^{2\pi} \left[\frac{1}{4}r^4 \cos\theta\sin\theta\right]_{r=0}^{r=1} d\theta$

$= 72\int_0^{2\pi} \frac{1}{4}\cos\theta\sin\theta\,d\theta = -9\sin^2\theta\Big|_0^{2\pi} = 0$

21. Let $u = x + y$ and $v = 2x - y$. Then $-1 \le u \le 3$ and $0 \le v \le 4$. Solving for x and y, we obtain $x = \frac{1}{3}(u + v)$ and $y = \frac{1}{3}(2u - v)$. Thus,

$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{vmatrix} = -\frac{1}{3}$,

so

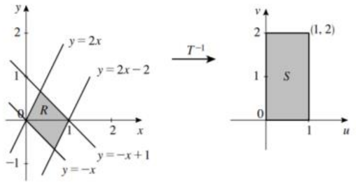


$\iint_R (2x + y)\,dA = \int_{-1}^3 \int_0^4 \left[\frac{2}{3}(u+v) + \frac{1}{3}(2u - v)\right] \left|-\frac{1}{3}\right| dv\,du = \frac{1}{9}\int_{-1}^3 \int_0^4 (4u + v)\,dv\,du$

$= \frac{1}{9}\int_{-1}^3 \left[4uv + \frac{1}{2}v^2\right]_{v=0}^{v=4} du = \frac{1}{9}\int_{-1}^3 (16u + 8)\,du = \frac{8}{9}\left(u^2 + u\right)\Big|_{-1}^3 = \frac{32}{3}$

22. Let $T : u = x + y,\ v = 2x - y$. Then $T^{-1} : x = \frac{1}{3}(u + v),\ y = \frac{1}{3}(2u - v)$. The parallelogram R is mapped onto the rectangular region S. Thus,

$$J = \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{vmatrix} = -\frac{1}{3},$$



so

$$\iint_R (x + y)\sin(2x - y)\,dA = \int_0^1 \int_0^2 (u \sin v)\left|-\frac{1}{3}\right| dv\,du = \frac{1}{3}\int_0^1 [-u\cos v]_0^2\,du = \frac{1}{3}\int_0^1 (1 - \cos 2)\,u\,du = \frac{1}{6}(1 - \cos 2).$$