6.4 General Exponential and Logarithmic Functions

24. $f(u) = 2^{u^2} \Rightarrow f'(u) = \ln 2 \left(2^{u^2} \cdot 2u\right) = (2\ln 2)\, u \cdot 2^{u^2}$

27. $f(x) = x^e + e^x \Rightarrow f'(x) = ex^{e-1} + e^x$

45. Let $u = x^2 + 2x$, so $du = 2(x+1)\,dx$. Then $\int (x+1)\,3^{x^2+2x}\,dx = \frac{1}{2}\int 3^u\,du = \dfrac{3^u}{2\ln 3} + C = \dfrac{3^{x^2+2x}}{2\ln 3} + C$.

48. Let $u = \sqrt{x}$, so $du = \dfrac{1}{2\sqrt{x}}\,dx$, $x = 1 \Rightarrow u = 1$, and $x = 4 \Rightarrow u = 2$. Thus, $\displaystyle\int_1^4 \frac{3^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int_1^2 3^u\,du = 2 \cdot \frac{3^u}{\ln 3}\bigg|_1^2 = \frac{12}{\ln 3}$.

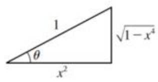
6.5 Inverse Trigonometric Functions

33. $f'(x) = \dfrac{d}{dx}\sin^{-1} 3x = \dfrac{1}{\sqrt{1-(3x)^2}} \cdot \dfrac{d}{dx}(3x) = \dfrac{3}{\sqrt{1-9x^2}}$

47. $g'(t) = \dfrac{d}{dt}\tan^{-1}\dfrac{t-1}{t+1} = \dfrac{\dfrac{d}{dt}\left(\dfrac{t-1}{t+1}\right)}{1+\left(\dfrac{t-1}{t+1}\right)^2} = \dfrac{\dfrac{(t+1)-(t-1)}{(t+1)^2}}{1+\dfrac{(t-1)^2}{(t+1)^2}} = \dfrac{2}{(t+1)^2+(t-1)^2} = \dfrac{1}{t^2+1}$

51. $f'(x) = \dfrac{d}{dx}\sin^{-1}\left(e^{2x}\right) = \dfrac{1}{\sqrt{1-\left(e^{2x}\right)^2}}\dfrac{d}{dx}e^{2x} = \dfrac{2e^{2x}}{\sqrt{1-e^{4x}}}$

53. Let $\theta = \cos^{-1} x^2$. Then $\cos\theta = x^2$, so



$h(x) = \cot\left(\cos^{-1} x^2\right) = \cot\theta = \dfrac{x^2}{\sqrt{1-x^4}} \Rightarrow$

$h'(x) = \dfrac{\left(1-x^4\right)^{1/2}(2x) - x^2\left(\frac{1}{2}\right)\left(1-x^4\right)^{-1/2}\left(-4x^3\right)}{1-x^4} = \dfrac{2x}{\left(1-x^4\right)^{3/2}}$.

~~ 参考作業,不交 ~~

6.7 Indeterminate Forms and l'Hôpital's Rule

18. $\displaystyle\lim_{x\to 1}\frac{x^{1/2}-x^{1/3}}{x-1} = \lim_{x\to 1}\frac{\frac{1}{2}x^{-1/2}-\frac{1}{3}x^{-2/3}}{1} = \frac{1}{2}-\frac{1}{3} = \frac{1}{6}$

19. $\displaystyle\lim_{x\to\infty}\frac{\ln(1+e^x)}{x^2} = \lim_{x\to\infty}\frac{e^x/(1+e^x)}{2x} = \lim_{x\to\infty}\frac{1}{(1/e^x+1)(2x)} = 0$

51. $\displaystyle\lim_{x\to\infty}(1+1/x)^{x^3} = \lim_{x\to\infty}\left[(1+1/x)^x\right]^{x^2} = \infty$. Compare with Example 10, in which we calculated $\displaystyle\lim_{x\to\infty}(1+1/x)^x = e$.

52. $\displaystyle\lim_{x\to\infty}(1-1/x)^x$ is an indeterminate form of type 1^∞. Let $y = (1-1/x)^x$, so $\ln y = \ln(1-1/x)^x = x\ln(1-1/x)$. Then

$\displaystyle\ln\left(\lim_{x\to\infty} y\right) = \lim_{x\to\infty}\ln y = \lim_{x\to\infty}\left[x\ln\left(1-\frac{1}{x}\right)\right] = \lim_{x\to\infty}\frac{\ln\left(1-\frac{1}{x}\right)}{\frac{1}{x}} = \lim_{x\to\infty}\frac{\frac{1/x^2}{1-1/x}}{-1/x^2} = \lim_{x\to\infty}\left(\frac{1}{1/x-1}\right) = -1$,

so $\displaystyle\lim_{x\to\infty} y = \lim_{x\to\infty}(1-1/x)^x = e^{-1} = 1/e$.