6.2 Inverse Functions

45. a. $f(x) = x^2$ on $[0, \infty) \Rightarrow g(x) = f^{-1}(x) = \sqrt{x}$. Using Formula 3, $g'(x) = \dfrac{1}{f'(g(x))} = \dfrac{1}{2y}\Big|_{y=\sqrt{x}} = \dfrac{1}{2\sqrt{x}}$.

b. From the result of part a, $g(x) = \sqrt{x}$, so $g'(x) = \dfrac{1}{2\sqrt{x}}$.

50. a. $f(x) = \dfrac{x+1}{2x-1} \Rightarrow f(1) = \dfrac{2}{1} = 2$, so $(1, 2)$ lies on the graph of f.

b. $f'(x) = \dfrac{(2x-1)-(x+1)(2)}{(2x-1)^2} = -\dfrac{3}{(2x-1)^2} \Rightarrow g'(2) = \dfrac{1}{f'(g(2))} = \dfrac{1}{f'(1)} = -\dfrac{(2x-1)^2}{3}\Big|_{x=1} = -\dfrac{1}{3}$

56. $H'(x) = \dfrac{d}{dx}[g(g(x))] = g'(g(x))\,g'(x)$, so
$H'(3) = g'(g(3))\,g'(3) = g'(4)\,g'(3) = \dfrac{1}{f'(g(4))} \cdot \dfrac{1}{f'(g(3))} = \dfrac{1}{f'(5)} \cdot \dfrac{1}{f'(4)} = \dfrac{1}{2} \cdot \dfrac{1}{\frac{1}{2}} = 1.$

57. Observe that $f(2) = \displaystyle\int_2^2 \frac{dt}{\sqrt{1+t^3}} = 0$, showing that $(2, 0)$ lies on the graph of f. Next, observe that $f'(x) = \dfrac{d}{dx}\displaystyle\int_2^x \frac{dt}{\sqrt{1+t^3}} = \frac{1}{\sqrt{1+x^3}}$ (by the FTC, Part 1). Therefore, $\left(f^{-1}\right)'(0) = \dfrac{1}{f'\left(f^{-1}(0)\right)} = \dfrac{1}{f'(2)} = \sqrt{1+2^3} = 3.$

6.3 Exponential Functions

25. $g'(x) = \dfrac{d}{dx}\left(\dfrac{e^{2x}}{1+e^{-x}}\right) = \dfrac{\left(1+e^{-x}\right)\left(2e^{2x}\right) - e^{2x}\left(-e^{-x}\right)}{\left(1+e^{-x}\right)^2} = \dfrac{2e^{2x}+2e^{x}+e^{x}}{\left(1+e^{-x}\right)^2} = \dfrac{e^{x}\left(2e^{x}+3\right)}{\left(1+e^{-x}\right)^2}$

36. $h'(x) = \dfrac{d}{dx}\left[\tan\left(e^{2x}+\ln x\right)\right] = \left[\sec^2\left(e^{2x}+\ln x\right)\right]\left(2e^{2x}+\dfrac{1}{x}\right) = \dfrac{1}{x}\left(2xe^{2x}+1\right)\sec^2\left(e^{2x}+\ln x\right)$

42. $e^{xy} - x^2 + y^2 = 5 \Rightarrow e^{xy}\left(y+xy'\right) - 2x + 2yy' = 0 \Leftrightarrow y'\left(xe^{xy}+2y\right) + ye^{xy} - 2x = 0 \Leftrightarrow y' = \dfrac{2x - ye^{xy}}{xe^{xy}+2y}$

49. $y = xe^{-x} \Rightarrow y' = e^{-x} - xe^{-x} = (1-x)e^{-x} \Rightarrow y'\big|_1 = 0$. Thus, the slope of the required tangent line is $m = 0$, and an equation of the line is $y - e^{-1} = 0(x-1)$ or $y = 1/e$.