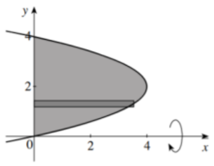
5.3 Volumes Using Cylindrical Shells

6. $V = 2\pi \int_0^2 (3-x)\left[\left(\frac{1}{2}x^2+2\right)-x^2\right]dx = 2\pi \int_0^2 \left(\frac{1}{2}x^3 - \frac{3}{2}x^2 - 2x + 6\right)dx = 2\pi \left(\frac{1}{8}x^4 - \frac{1}{2}x^3 - x^2 + 6x\right)\Big|_0^2 = 12\pi$

12. $V = 2\pi \int_0^4 yx\,dy = 2\pi \int_0^4 y\left(-y^2+4y\right)dy$

$= 2\pi \int_0^4 \left(-y^3+4y^2\right)dy = 2\pi\left(-\frac{1}{4}y^4+\frac{4}{3}y^3\right)\Big|_0^4$

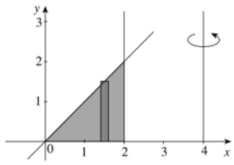
$= \frac{128\pi}{3}$



29. $V = 2\pi \int_0^2 (4-x)\,x\,dx$

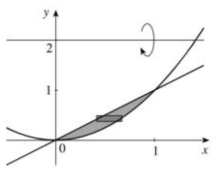
$= 2\pi \int_0^2 \left(4x - x^2\right)dx$

$= 2\pi\left(2x^2 - \frac{1}{3}x^3\right)\Big|_0^2 = \frac{32\pi}{3}$



34. $V = 2\pi \int_0^1 (2-y)\left(\sqrt{y}-y\right)dy = 2\pi \int_0^1 \left(y^2 - y^{3/2} - 2y + 2y^{1/2}\right)dy$

$= 2\pi\left(\frac{1}{3}y^3 - \frac{2}{5}y^{5/2} - y^2 + \frac{4}{3}y^{3/2}\right)\Big|_0^1 = \frac{8\pi}{15}$



6.1 The Natural Logarithmic Function

34. $f(x) = \ln\left(x+\sqrt{x^2-1}\right) \Rightarrow$

$$f'(x) = \frac{1+\frac{1}{2}\left(x^2-1\right)^{-1/2}(2x)}{x+\left(x^2-1\right)^{1/2}} = \frac{1+x\left(x^2-1\right)^{-1/2}}{x+\left(x^2-1\right)^{1/2}} = \frac{1+\dfrac{x}{\sqrt{x^2-1}}}{x+\sqrt{x^2-1}} = \frac{\dfrac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}}}{x+\sqrt{x^2-1}} = \frac{1}{\sqrt{x^2-1}}$$

37. $f'(x) = \dfrac{d}{dx}[\ln(\ln x)] = \dfrac{\dfrac{d}{dx}(\ln x)}{\ln x} = \dfrac{1}{x\ln x}$

49. $\ln y - x\ln x = -1 \Rightarrow \dfrac{1}{y}\cdot y' - \left(\ln x + x\cdot\dfrac{1}{x}\right) = 0 \Rightarrow y' = y(\ln x + 1)$

56. $y - \ln\left(x^2+y^2\right) = 0 \Rightarrow y' - \dfrac{2x+2yy'}{x^2+y^2} = 0$. Substituting $x = 1$ and $y = 0$ into the equation gives $y' - \dfrac{2+0}{1+0} = 0$ or $y' = 2$, the slope of the required tangent line. An equation is $y - 0 = 2(x-1)$ or $y = 2x - 2$.