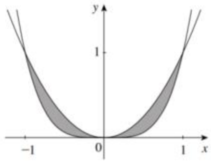
5.1 Areas Between Curves

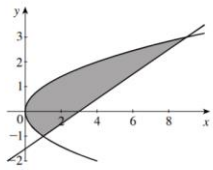
16. Solve $x^2 = x^4 \Leftrightarrow x^4 - x^2 = x^2(x-1)(x+1) = 0$, giving $(-1, 1)$, $(0, 0)$, and $(1, 1)$ as the points of intersection. By symmetry,

$$A = 2\int_0^1 \left(x^2 - x^4\right) dx = 2\left(\tfrac{1}{3}x^3 - \tfrac{1}{5}x^5\right)\Big|_0^1 = \tfrac{4}{15}.$$



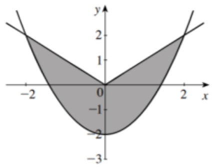
28. Solve $y^2 = 2y + 3 \Leftrightarrow y^2 - 2y - 3 = (y-3)(y+1) = 0$, giving $(1, -1)$ and $(9, 3)$ as the points of intersection. Thus,

$$A = \int_{-1}^3 \left[(2y+3) - y^2\right] dy = \int_{-1}^3 \left(-y^2 + 2y + 3\right) dy$$
$$= -\tfrac{1}{3}y^3 + y^2 + 3y\Big|_{-1}^3 = \tfrac{32}{3}$$



31. Solve $|x| = x^2 - 2$ to find the points of intersection of the two curves. Since the graphs of both functions are symmetric with respect to the y-axis, it suffices to find the positive solution of $x = x^2 - 2 \Leftrightarrow$ $x^2 - x - 2 = (x-2)(x+1) = 0$, giving 2 as the only root (we reject -1, because $x > 0$). By symmetry, the points of intersection are $(-2, 2)$ and $(2, 2)$.

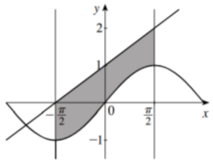
$$A = 2\int_0^2 \left[|x| - \left(x^2 - 2\right)\right] dx = 2\int_0^2 \left(-x^2 + x + 2\right) dx$$
$$= 2\left(-\tfrac{1}{3}x^3 + \tfrac{1}{2}x^2 + 2x\right)\Big|_0^2 = \tfrac{20}{3}$$



32. $A = \int_{-\pi/2}^{\pi/2} \left[\left(\tfrac{2}{\pi}x + 1\right) - \sin x\right] dx$

$= 2\int_0^{\pi/2} 1\, dx = \pi$

because $f(x) = \frac{2}{\pi}x - \sin x$ is odd.



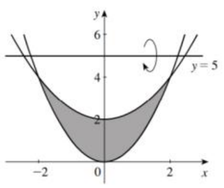
5.2 Volumes: Disks, Washers, and Cross-Sections

11. $V = \pi\int_0^1 \left[\left(\tfrac{3}{2} - x^3\right)^2 - \left(\tfrac{3}{2} - x\right)^2\right] dx = \pi\int_0^1 \left(x^6 - 3x^3 - x^2 + 3x\right) dx = \pi\left(\tfrac{1}{7}x^7 - \tfrac{3}{4}x^4 - \tfrac{1}{3}x^3 + \tfrac{3}{2}x^2\right)\Big|_0^1 = \tfrac{47\pi}{84}$

12. $V = \pi\int_0^1 \left[\left(\tfrac{3}{2} - y\right)^2 - \left(\tfrac{3}{2} - y^{1/3}\right)^2\right] dy = \pi\int_0^1 \left(y^2 - 3y - y^{2/3} + 3y^{1/3}\right) dy = \pi\left(\tfrac{1}{3}y^3 - \tfrac{3}{2}y^2 - \tfrac{3}{5}y^{5/3} + \tfrac{9}{4}y^{4/3}\right)\Big|_0^1$
$= \tfrac{29\pi}{60}$

36. To find the points of intersection of the two graphs, we solve $x^2 = \frac{1}{2}x^2 + 2$, giving $(-2, 4)$ and $(2, 4)$.

$$V = \pi\int_{-2}^2 \left\{\left(5 - x^2\right)^2 - \left[5 - \tfrac{1}{2}x^2 - 2\right]^2\right\} dx$$
$$= 2\pi\int_0^2 \left[\left(x^2 - 5\right)^2 - \left(\tfrac{1}{2}x^2 - 3\right)^2\right] dx$$
$$= 2\pi\int_0^2 \left(\tfrac{3}{4}x^4 - 7x^2 + 16\right) dx$$
$$= 2\pi\left(\tfrac{3}{20}x^5 - \tfrac{7}{3}x^3 + 16x\right)\Big|_0^2 = \tfrac{544\pi}{15}$$



37. $V = \pi\int_1^3 \left\{(-1-2)^2 - \left[-1 - \left(y^2 - 4y + 5\right)\right]^2\right\} dy$

$= \pi\int_1^3 \left[9 - \left(y^2 - 4y + 6\right)^2\right] dy$

$= \pi\int_1^3 \left(-y^4 + 8y^3 - 28y^2 + 48y - 27\right) dy$

$= \pi\left(-\tfrac{1}{5}y^5 + 2y^4 - \tfrac{28}{3}y^3 + 24y^2 - 27y\right)\Big|_1^3 = \tfrac{104\pi}{15}$