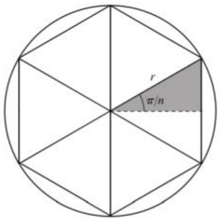
4.3 Area

31. $\sum_{k=1}^{10}(2k+1) = 2\sum_{k=1}^{10} k + \sum_{k=1}^{10} 1 = 2\left[\frac{10(11)}{2}\right] + 10 = 120$

43.
$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)^2\left(\frac{1}{n}\right) = \lim_{n\to\infty}\left(\frac{1}{n}\right)\left(\sum_{k=1}^{n}1 + \frac{4}{n}\sum_{k=1}^{n}k + \frac{4}{n^2}\sum_{k=1}^{n}k^2\right)$$
$$= \lim_{n\to\infty}\left[\frac{1}{n}\cdot n + \frac{4}{n^2}\cdot\frac{n(n+1)}{2} + \frac{4}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right]$$
$$= \lim_{n\to\infty}\left[1 + 2\left(1+\frac{1}{n}\right) + \frac{2}{3}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right] = \frac{13}{3}$$

57. **a.** The area of the shaded triangle is $\frac{1}{2}$ (base) (height) $= \frac{1}{2}\left(r\cos\frac{\pi}{n}\right)\left(r\sin\frac{\pi}{n}\right)$, so the area of each isosceles triangle is $2\cdot\frac{1}{2}r^2\cos\frac{\pi}{n}\sin\frac{\pi}{n} = \frac{1}{2}r^2\sin\frac{2\pi}{n}$. Therefore, $A_n = \frac{1}{2}r^2 n\sin\frac{2\pi}{n}$.



b. $A = \lim_{n\to\infty}A_n = \lim_{n\to\infty}\frac{1}{2}r^2 n\sin\frac{2\pi}{n} = \frac{1}{2}r^2\lim_{n\to\infty}n\left(\frac{2\pi}{2\pi}\right)\sin\frac{2\pi}{n}$
$= \frac{1}{2}r^2(2\pi)\lim_{n\to\infty}\frac{\sin\frac{2\pi}{n}}{\frac{2\pi}{n}} = \pi r^2$

58. **a.** Refer to the figure in Exercise 57. The length of each side of the isosceles triangle is $2r\sin\frac{\pi}{n}$, so the perimeter of the polygon is $C_n = 2nr\sin\frac{\pi}{n}$.

b. $C = \lim_{n\to\infty}C_n = \lim_{n\to\infty}2nr\sin\frac{\pi}{n} = 2r\lim_{n\to\infty}n\left(\frac{\pi}{\pi}\right)\sin\frac{\pi}{n} = 2\pi r\lim_{n\to\infty}\frac{\sin\frac{\pi}{n}}{\frac{\pi}{n}} = 2\pi r$

4.5 The Fundamental Theorem of Calculus

9. $g'(x) = \frac{d}{dx}\int_2^{\sqrt{x}}\frac{\sin t}{t}\,dt = \frac{\sin\sqrt{x}}{\sqrt{x}}\cdot\frac{d}{dx}(\sqrt{x}) = \frac{\sin\sqrt{x}}{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}} = \frac{\sin\sqrt{x}}{2x}$

19. $\int_1^4\frac{1}{\sqrt{x}}\,dx = \int_1^4 x^{-1/2}\,dx = 2\sqrt{x}\Big|_1^4 = 4-2 = 2$

42. Let $u = \cos\theta$, so $du = -\sin\theta\,d\theta \Rightarrow \sin\theta\,d\theta = -du$, $\theta = 0 \Rightarrow u = 1$, and $\theta = \frac{\pi}{2} \Rightarrow u = 0$. Then $\int_0^{\pi/2}\sqrt{\cos\theta}\sin\theta\,d\theta = -\int_1^0 u^{1/2}\,du = \int_0^1 u^{1/2}\,du = \frac{2}{3}u^{3/2}\Big|_0^1 = \frac{2}{3}$.

88. $\frac{d}{dy}\left[\int_0^x\sqrt{3+2\cos t}\,dt + \int_0^y\sin t\,dt\right] = \frac{d}{dy}(0) = 0 \Rightarrow \sqrt{3+2\cos x}\frac{dx}{dy} + \sin y = 0 \Rightarrow \frac{dx}{dy} = -\frac{\sin y}{\sqrt{3+2\cos x}}$