3.5 Limits Involving Infinity and Asymptotes

21. $\lim_{x\to-\infty}\frac{3x+4}{2x-3}=\lim_{x\to-\infty}\frac{3+\frac{4}{x}}{2-\frac{3}{x}}=\frac{3}{2}$

24. $\lim_{x\to-\infty}\frac{2x^3+x^2+3}{x+1}=\lim_{x\to-\infty}\frac{2x^2\left(1+\frac{1}{2x}+\frac{3}{2x^3}\right)}{1+\frac{1}{x}}=\infty$

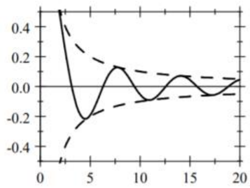
25. $\lim_{x\to\infty}\left(\frac{x^3}{3x^2-2}-\frac{x^2}{3x+1}\right)=\lim_{x\to\infty}\left[\frac{x^3(3x+1)-x^2\left(3x^2-2\right)}{(3x^2-2)(3x+1)}\right]=\lim_{x\to\infty}\frac{x^3+2x^2}{(3x^2-2)(3x+1)}$

$=\lim_{x\to\infty}\frac{1+\frac{2}{x}}{\left(3-\frac{2}{x^2}\right)\left(3+\frac{1}{x}\right)}=\frac{1}{9}$

39. a. Since $|\sin x|\le 1 \iff -1\le \sin x\le 1$, we have $-\frac{1}{x}\le\frac{\sin x}{x}\le\frac{1}{x}$ for $x>0$.

b. Noting that $\lim_{x\to\infty}\frac{1}{x}=0$ and applying the Squeeze Theorem to the inequalities in part a, we obtain $\lim_{x\to\infty}\frac{\sin x}{x}=0$.

c.



4.1 Indefinite Integrals

9. $\int t^{1/3}(t-1)^2\,dt=\int\left(t^{7/3}-2t^{4/3}+t^{1/3}\right)dt=\frac{3}{10}t^{10/3}-\frac{6}{7}t^{7/3}+\frac{3}{4}t^{4/3}+C$

20. $\int\left(\csc\theta\cot\theta-3\sec^2\theta\right)d\theta=-\csc\theta-3\tan\theta+C$

38. $f(x)=\int f'(x)\,dx=\int\left(1+x^{-2}\right)dx=x-\frac{1}{x}+C$. $f(1)=2\Rightarrow 1-1+C=2\Rightarrow C=2$. Thus, $f(x)=x-\frac{1}{x}+2$.

42. $f'(x)=\int f''(x)\,dx=\int(2x+1)\,dx=x^2+x+C_1$. $f'(0)=1\Rightarrow 0+C_1=1$, so $f'(x)=x^2+x+1$. Then $f(x)=\int f'(x)\,dx=\int\left(x^2+x+1\right)dx=\frac{1}{3}x^3+\frac{1}{2}x^2+x+C_2$. $f(0)=5\Rightarrow 0+C_2=5$ or $C_2=5$. Thus, $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2+x+5$.

4.2 Integration by Substitution

13. For $I=\int\left(x^2+x-1\right)^3(2x+1)\,dx$, let $u=x^2+x-1\Rightarrow du=(2x+1)\,dx$. Then $I=\int u^3\,du=\frac{1}{4}u^4+C=\frac{1}{4}\left(x^2+x-1\right)^4+C$.

31. For $I=\int\sin\pi x\cos\pi x\,dx$, let $u=\cos\pi x\Rightarrow du=-\pi\sin\pi x\,dx\Rightarrow \sin\pi x\,dx=-\frac{du}{\pi}$. Then $I=-\frac{1}{\pi}\int u\,du=-\frac{1}{2\pi}u^2+C=-\frac{1}{2\pi}\cos^2\pi x+C$. If we let $u=\sin\pi x$, then $I=\frac{1}{2\pi}\sin^2\pi x+C$.

43. For $I=\int\sin^2\pi x\,dx=\int\frac{1-\cos 2\pi x}{2}\,dx=\int\frac{1}{2}\,dx-\frac{1}{2}\int\cos 2\pi x\,dx$, let $u=2\pi x$ in the second integral. Then $I=\frac{1}{2}x-\frac{1}{2}\cdot\frac{1}{2\pi}\int\cos u\,du=\frac{1}{2}x-\frac{1}{4\pi}\sin u+C=\frac{1}{2}x-\frac{1}{4\pi}\sin 2\pi x+C$.

50. For $I=\int\frac{\sqrt{a^2-x^2}}{x^4}\,dx$, let $x=\frac{1}{t}\Rightarrow dx=-\frac{1}{t^2}\,dt$ and $t=\frac{1}{x}$. Then

$I=\int\frac{\sqrt{a^2-(1/t^2)}}{(1/t)^4}\left(-\frac{1}{t^2}\right)dt=-\int t\sqrt{a^2t^2-1}\,dt$. Now let $u=a^2t^2-1\Rightarrow du=2a^2t\,dt$, so

$I=-\frac{1}{2a^2}\int u^{1/2}\,du=-\frac{1}{2a^2}\left(\frac{2}{3}u^{3/2}\right)+C=-\frac{1}{3a^2}\left(a^2t^2-1\right)^{3/2}+C=-\frac{1}{3a^2}\left(\frac{a^2}{x^2}-1\right)^{3/2}+C$

$=-\frac{\left(a^2-x^2\right)^{3/2}}{3a^2x^3}+C.$