2.6 The Chain Rule

5. $y = \sqrt{x + \cos x} = g(f(x))$, where $u = f(x) = x + \cos x$ and $y = g(u) = \sqrt{u} = u^{1/2}$, so
$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \tfrac{1}{2}u^{-1/2}(1 - \sin x) = \frac{1 - \sin x}{2\sqrt{x + \cos x}}.$$

14. $h(x) = (2x-1)^2\left(x^2+1\right)^3 \Rightarrow$
$$h'(x) = (2x-1)^2 \cdot 3\left(x^2+1\right)^2(2x) + 2(2x-1)\cdot 2\left(x^2+1\right)^3 = 2(2x-1)\left(x^2+1\right)^2\left[3x(2x-1) + 2\left(x^2+1\right)\right]$$
$$= 2(2x-1)\left(x^2+1\right)^2\left(8x^2 - 3x + 2\right)$$

47. $y = \sin^2\left(\frac{1+x}{1-x}\right) \Rightarrow$
$$\frac{dy}{dx} = 2\sin\left(\frac{1+x}{1-x}\right)\cos\left(\frac{1+x}{1-x}\right)\frac{d}{dx}\left(\frac{1+x}{1-x}\right) = 2\sin\left(\frac{1+x}{1-x}\right)\cos\left(\frac{1+x}{1-x}\right)\cdot\frac{(1-x)(1)-(1+x)(-1)}{(1-x)^2}$$
$$= \frac{4}{(1-x)^2}\sin\left(\frac{1+x}{1-x}\right)\cos\left(\frac{1+x}{1-x}\right)$$

48. $y = \sec^3\left(\frac{\sqrt{x}}{1+x}\right) \Rightarrow$
$$\frac{dy}{dx} = 3\sec^2\left(\frac{\sqrt{x}}{1+x}\right)\sec\left(\frac{\sqrt{x}}{1+x}\right)\tan\left(\frac{\sqrt{x}}{1+x}\right)\frac{d}{dx}\left(\frac{\sqrt{x}}{1+x}\right)$$
$$= 3\sec^3\left(\frac{\sqrt{x}}{1+x}\right)\tan\left(\frac{\sqrt{x}}{1+x}\right)\cdot\frac{(1+x)\frac{1}{2\sqrt{x}} - \sqrt{x}(1)}{(1+x)^2} = 3\sec^3\left(\frac{\sqrt{x}}{1+x}\right)\tan\left(\frac{\sqrt{x}}{1+x}\right)\cdot\frac{1+x-2x}{2\sqrt{x}(1+x)^2}$$
$$= \frac{3(1-x)}{2\sqrt{x}(1+x)^2}\sec^3\left(\frac{\sqrt{x}}{1+x}\right)\tan\left(\frac{\sqrt{x}}{1+x}\right)$$

2.7 Implicit Differentiation

3. $xy^2 + yx^2 - 2 = 0 \Rightarrow y^2 + 2xyy' + y'x^2 + 2xy = 0 \Rightarrow \left(2xy + x^2\right)y' = -\left(y^2 + 2xy\right) \Rightarrow y' = -\frac{y(y+2x)}{x(2y+x)}$

15. $y^2 = \sin(x+y) \Rightarrow 2yy' = \cos(x+y)\cdot(1+y') \Rightarrow [2y - \cos(x+y)]\,y' = \cos(x+y) \Rightarrow y' = \frac{\cos(x+y)}{2y - \cos(x+y)}$

23. $x^{2/3} + y^{2/3} = 2 \Rightarrow \frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}y' = 0 \Rightarrow y' = -\frac{y^{1/3}}{x^{1/3}}$, so $y'|_{(1,-1)} = 1$. An equation of the tangent line is

29. $xy + x^3 = 4 \Rightarrow y + xy' + 3x^2 = 0 \Rightarrow y' = -\frac{3x^2 + y}{x}$. Differentiating both sides of the next-to-last expression yields
$$y' + y' + xy'' + 6x = 0 \Rightarrow y'' = -\frac{6x + 2y'}{x} = -\frac{6x - 2\cdot\frac{3x^2+y}{x}}{x} = -\frac{2\left(3x^2 - 3x^2 - y\right)}{x^2} = \frac{2y}{x^2}.$$