2.3 The Product and Quotient Rules

13. $F(x) = (x+2)\left(x^2 - x + 1\right) = x^3 - x^2 + x + 2x^2 - 2x + 2 = x^3 + x^2 - x + 2 \Rightarrow F'(x) = 3x^2 + 2x - 1.$

Using the Product Rule, we have

$$\begin{aligned} F'(x) &= (x+2)\frac{d}{dx}\left(x^2 - x + 1\right) + \left(x^2 - x + 1\right)\frac{d}{dx}(x+2) = (x+2)(2x-1) + \left(x^2 - x + 1\right)(1) \\ &= 2x^2 - x + 4x - 2 + x^2 - x + 1 = 3x^2 + 2x - 1 \end{aligned}$$

23.
$$\begin{aligned} f'(x) &= \frac{(x-2)\frac{d}{dx}\left(x^3 + x^2 + x + 1\right) - \left(x^3 + x^2 + x + 1\right)\frac{d}{dx}(x-2)}{(x-2)^2} \\ &= \frac{(x-2)\left(3x^2 + 2x + 1\right) - \left(x^3 + x^2 + x + 1\right)}{(x-2)^2} = \frac{3x^3 + 2x^2 + x - 6x^2 - 4x - 2 - x^3 - x^2 - x - 1}{(x-2)^2} \\ &= \frac{2x^3 - 5x^2 - 4x - 3}{(x-2)^2} \end{aligned}$$

55. $f(x) = x^{-1} + 3x^{-2} \Rightarrow f'(x) = -x^{-2} - 6x^{-3} \Rightarrow f''(x) = 2x^{-3} + 18x^{-4} = \frac{2}{x^3} + \frac{18}{x^4}$

61. **a.** $f(x) = 4x^3 - 2x^2 + 3 \Rightarrow f'(x) = 12x^2 - 4x \Rightarrow f''(x) = 24x - 4$, so $f''(2) = 24(2) - 4 = 44$.

b. $y = 2x^3 - \frac{1}{x} = 2x^3 - x^{-1} \Rightarrow y' = 6x^2 + x^{-2} \Rightarrow y'' = 12x - 2x^{-3}$, so $y''\big|_{x=1} = 12 - 2 = 10$.

2.5 Derivatives of Trigonometric Functions

11. $g'(x) = \frac{d}{dx}\left(\frac{\sin x}{x}\right) = \frac{x\cos x - \sin x}{x^2}$

17.
$$\begin{aligned} f'(x) &= \frac{d}{dx}\left(\frac{1+\sin x}{1-\cos x}\right) = \frac{(1-\cos x)(\cos x) - (1+\sin x)(\sin x)}{(1-\cos x)^2} = \frac{\cos x - \cos^2 x - \sin x - \sin^2 x}{(1-\cos x)^2} \\ &= \frac{\cos x - \sin x - 1}{(1-\cos x)^2} \end{aligned}$$

35. $y' = \frac{d}{dx}\left(\frac{\sin x}{1-\cos x}\right) = \frac{(1-\cos x)(\cos x) - \sin x(\sin x)}{(1-\cos x)^2} = \frac{1}{\cos x - 1} \Rightarrow y'\big|_{x=\pi/2} = -1$

41. $f(x) = \sin x \Rightarrow f'(x) = \cos x \Rightarrow f''(x) = -\sin x \Rightarrow f'''(x) = -\cos x \Rightarrow f^{(4)}(x) = \sin x \Rightarrow \cdots$. So higher order derivatives of $f(x)$ are either $\pm\sin x$ or $\pm\cos x$. Since $-1 \le \sin x \le 1$ and $-1 \le \cos x \le 1$ for all x, we see that $\left|f^{(n)}(x)\right| \le 1$ for all n and all x.