2.1 The Derivative

11. $f(x) = \dfrac{1}{x+2} \Rightarrow$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\dfrac{1}{x+h+2} - \dfrac{1}{x+2}}{h} = \lim_{h\to 0} \frac{\dfrac{x+2-(x+h+2)}{(x+2)(x+h+2)}}{h}$$

$$= \lim_{h\to 0} \frac{-h}{h(x+2)(x+h+2)} = -\lim_{h\to 0} \frac{1}{(x+2)(x+h+2)} = -\frac{1}{(x+2)^2}$$ with domain $(-\infty, -2) \cup (-2, \infty)$.

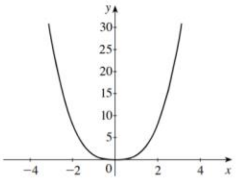
15. $f(x) = x^2 + 1 \Rightarrow$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\left[(x+h)^2 + 1\right] - \left(x^2+1\right)}{h} = \lim_{h\to 0} \frac{\left(x^2 + 2xh + h^2 + 1\right) - \left(x^2+1\right)}{h}$$

$$= \lim_{h\to 0} \frac{h(2x+h)}{h} = \lim_{h\to 0} (2x+h) = 2x$$

The slope of the tangent line at $(2, 5)$ is $f'(2) = 2(2) = 4$. An equation of the tangent line is $y - 5 = 4(x-2)$ or $y = 4x - 3$.

57. a.



b. $f(x) = \begin{cases} -x^3 & \text{if } x < 0 \\ x^3 & \text{if } x \geq 0 \end{cases}$ If $h < 0$, then

$\displaystyle\lim_{h\to 0^-} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0^-} \frac{-h^3}{h} = -\lim_{h\to 0^-} h^2 = 0$. If $h > 0$, then

$\displaystyle\lim_{h\to 0^+} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0^+} \frac{h^3}{h} = \lim_{h\to 0^+} h^2 = 0$. Therefore,

$\displaystyle\lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = 0$, and f is differentiable at 0.

If $x > 0$, then $f(x) = x^3$ and

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h\to 0} \frac{\left(x^3 + 3x^2h + 3xh^2 + h^3\right) - x^3}{h}$$

$$= \lim_{h\to 0} \frac{h\left(3x^2 + 3xh + h^2\right)}{h} = \lim_{h\to 0} \left(3x^2 + 3xh + h^2\right) = 3x^2$$

If $x < 0$, then $f(x) = -x^3$, and a similar calculation shows that $f'(x) = -3x^2$. So f is differentiable everywhere.

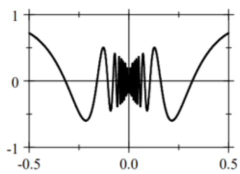
c. From the results of part b, we see that $f'(x) = \begin{cases} -3x^2 & \text{if } x < 0 \\ 3x^2 & \text{if } x \geq 0 \end{cases}$

60. a. Observe that $0 \leq \left|x^{1/3} \sin \frac{1}{x}\right| \leq |x|^{1/3}$, so the Squeeze Theorem implies that $\lim_{x\to 0} x^{1/3} \sin \frac{1}{x} = 0 = f(0)$. This shows that f is continuous at 0. Next,

$$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{h^{1/3} \sin \frac{1}{h}}{h} = \lim_{h\to 0} \frac{\sin \frac{1}{h}}{h^{2/3}}$$

does not exist, and so f is not differentiable.

b.



2.2 Basic Rules of Differentiation

17. $g'(x) = \dfrac{d}{dx}\left[x^2\left(2x^3 - 3x^2 + x + 4\right)\right] = \dfrac{d}{dx}\left(2x^5 - 3x^4 + x^3 + 4x^2\right) = 10x^4 - 12x^3 + 3x^2 + 8x$

31. $y' = \dfrac{d}{dx}\left(x^{1/3} + x^{-1/2}\right) = \frac{1}{3}x^{-2/3} - \frac{1}{2}x^{-3/2} = \dfrac{1}{3x^{2/3}} - \dfrac{1}{2x^{3/2}}$

50. The line $y = 2x$ has slope 2. Also $y = x^2 + c \Rightarrow \dfrac{dy}{dx} = 2x$, so $2x = 2 \Rightarrow x = 1$. Therefore $y = 2$. Substituting into the second equation gives $2 = 1 + c$, so $c = 1$.

53. $\displaystyle\lim_{h\to 0} \frac{(1+h)^3 - 1}{h} = f'(1)$, where $f(x) = x^3$. $f'(x) = 3x^2$, so $f'(1) = 3$.