Homework-13

Exercises of Section(6.3 - 6.4)

2018.01.09

- **(Section 6.3)**

(11). Differentiate the function.

$$\boldsymbol{f(x) = \sqrt{e^x + e^{-x}}}$$

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○ Sol :

$$\frac{d}{dx}f(x) = \frac{1}{2}(e^x + e^{-x})^{-1/2} \cdot (e^x - e^{-x}) = \frac{e^x - e^{-x}}{2\sqrt{e^x + e^{-x}}}$$

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(42). Use implicit differentiation to find dy/dx

$$\boldsymbol{e^{xy} - x^2 + y^2 = 5}$$

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○ Sol : Differentiate both sides with respect to x

$$e^{xy} \cdot (y + xy') - 2x + 2y \cdot y' = 0 \;\Rightarrow\; (xe^{xy} + 2y) \cdot y' = 2x - ye^{xy} \;\Rightarrow\; y' = \frac{2x - ye^{xy}}{xe^{xy} + 2y}$$

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(43). Use implicit differentiation to find dy/dx

$$\boldsymbol{e^x \sec y - xy^2 = 0}$$

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○ Sol : Differentiate both sides with respect to x

$$e^x \sec y + e^x \sec y \tan y \cdot y' - y^2 - 2xy \cdot y' = 0$$

$$\Rightarrow \quad (e^x \sec y \tan y - 2xy)y' = y^2 - e^x \sec y$$

$$\Rightarrow \quad y' = \frac{y^2 - e^x \sec y}{e^x \sec y \tan y - 2xy}$$

(50). Find an equation of the tangent line to the curve of $xe^y + 2x + y = 3$ at $(1, 0)$.

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∘ Sol : Use implicit differentiation to find dy/dx

$$e^y + xe^y \cdot y' + 2 + y' = 0 \ \Rightarrow \ y' = \frac{-2 - e^y}{1 + xe^y}$$

At point $(1, 0)$ we have

$$y'\Big|_{x=1, y=0} = \frac{-3}{2}$$

Hence we have the equation of the tangent line

$$y - 0 = -\frac{3}{2}(x - 1) \ \Rightarrow \ y = -\frac{3}{2}x + \frac{3}{2}$$

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(51). Find the absolute extrema of the function on the indicated interval.

$$\boldsymbol{f(x) = xe^{-x}} \ , \ [\mathbf{-1, 2}]$$

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∘ Sol :

$$f'(x) = e^{-x} - xe^{-x} = (1 - x)e^{-x} \ \Rightarrow \ f'(x) = 0 \ \Rightarrow \ x = 1$$

Hence $x = 1$ is an critical point.

Compare with values of critical point and end points we have :

$x =$	-1	1	2
$f(x) =$	$-e$	$\frac{1}{e}$	$\frac{1}{2e^2}$

Note that $-e < \frac{1}{2e^2} < \frac{1}{e}$ then we have

$$\begin{cases} \text{absolute maximum value} \quad 1/e \text{ at } x = 1 \\ \text{absolute minimum value} \quad -e \text{ at } x = -1 \end{cases}$$

• (Section 6.4)

(23). Differentiate the function

$$\boldsymbol{y = x(5^{3x})}$$

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∘ Sol :

$$y'(x) = 5^{3x} + x \cdot \frac{d}{dx}(5^{3x}) = 5^{3x} + x \cdot 5^{3x} \cdot (3\ln 5) = 5^{3x}\Big(1 + 3x\ln 5\Big)$$

(30). Differentiate the function

$$\boldsymbol{h(x) = 2^{\tan x}}$$

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◦ Sol :

$$h'(x) = \frac{d}{dx}\ e^{\tan x \cdot \ln 2} = 2^{\tan x} \cdot \left(\sec^2 x \cdot \ln 2\right) = \ln 2 \cdot \sec^2 x \cdot 2^{\tan x}$$

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(41). Use logarithmic differentiation to find the derivative of the function

$$\boldsymbol{y = (\sqrt{\cos x})^x}$$

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◦ Sol

$$\begin{aligned} y = (\sqrt{\cos x})^x \qquad & \Rightarrow\ \ln y = x \ln \sqrt{\cos x} \\ & \Rightarrow\ \frac{y'}{y} = \ln \sqrt{\cos x} + x \cdot \frac{-\frac{1}{2}(\cos x)^{-1/2} \cdot \sin x}{\sqrt{\cos x}} \\ & \Rightarrow\ \frac{y'}{y} = \ln \sqrt{\cos x} - \frac{x \sin x}{2 \cos x} = \frac{\cos x \ln \cos x - x \sin x}{2 \cos x} \\ & \Rightarrow\ y' = \frac{\cos x \ln \cos x - x \sin x}{2 \cos x} \cdot (\sqrt{\cos x})^x \end{aligned}$$

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(45). Evaluate the integral :

$$\boldsymbol{\int (x+1) 3^{x^2+2x}\ dx}$$

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◦ Sol :

Let $u = x^2 + 2x \ \Rightarrow\ du = (2x+2)\ dx = 2(x+1)dx$

$$\int (x+1)3^{x^2+2x}\ dx = \frac{1}{2}\int 2(x+1)3^{x^2+2x}\ dx = \frac{1}{2}\int 3^u\ du = \frac{1}{2}\left(\frac{3^u}{\ln 3}\right) + C = \frac{3^{x^2+2x}}{2\ln 3} + C$$

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(46). Evaluate the integral :

$$\boldsymbol{\int_0^1 (3^t + t^3) dt}$$

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◦ Sol :

$$\int_0^1 (3^t + t^3)dt = \left(\frac{3^t}{\ln 3} + \frac{1}{4}t^4\right)\Bigg|_0^1 = \frac{3}{\ln 3} + \frac{1}{4} - \frac{1}{\ln 3} = \frac{1}{4} + \frac{2}{\ln 3}$$
