**5.** $u = \tan x \Rightarrow du = \sec^2 x\, dx$, so $\int \tan^3 x \sec^2 x\, dx = \int u^3 du = \frac{1}{4}u^4 + C = \frac{1}{4}\tan^4 x + C.$

**20.** For $I = \displaystyle\int \frac{x^2 + \frac{2}{3}}{\left(x^3 + 2x\right)^2}\, dx$, let $u = x^3 + 2x \Rightarrow du = \left(3x^2 + 2x\right) dx = 3\left(x^2 + \frac{2}{3}\right) dx$. Then

$$I = \frac{1}{3}\int \frac{du}{u^2} = \frac{1}{3}\int u^{-2}\, du = \frac{1}{3}\left(-\frac{1}{u}\right) + C = -\frac{1}{3x\left(x^2+2\right)} + C.$$

**34.** For $I = \int \sqrt{\sin\theta}\cos\theta\, d\theta$, let $u = \sin\theta \Rightarrow du = \cos\theta\, d\theta$. Then $I = \int \sqrt{u}\, du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}\sqrt{\sin^3\theta} + C.$

**42.** For $I = \int \sec^2(x+1)\sqrt{1+\tan(x+1)}\, dx$, let $u = 1 + \tan(x+1) \Rightarrow du = \sec^2(x+1)\, dx$. Then $I = \int u^{1/2}\, du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}\sqrt{[1+\tan(x+1)]^3} + C.$

**34.** $\sum_{k=1}^{40} k\left(k^2 - k\right) = \sum_{k=1}^{40} k^3 - \sum_{k=1}^{40} k^2 = \left[\frac{40(40+1)}{2}\right]^2 - \frac{40(40+1)(2\cdot 40+1)}{6} = 650{,}260$

**38.**
$$\sum_{k=1}^{n}\frac{1}{n}\left(1+\frac{k}{n}\right)^2 = \sum_{k=1}^{n}\frac{1}{n}\left(1+\frac{2k}{n}+\frac{k^2}{n^2}\right) = \frac{1}{n}\left(\sum_{k=1}^{n}1 + \frac{2}{n}\sum_{k=1}^{n}k + \frac{1}{n^2}\sum_{k=1}^{n}k^2\right)$$
$$= \frac{1}{n}\left[n + \frac{2}{n}\cdot\frac{n(n+1)}{2} + \frac{1}{n^2}\cdot\frac{n(n+1)(2n+1)}{6}\right] = \frac{14n^2+9n+1}{6n^2}$$

**42.**
$$\lim_{n\to\infty}\sum_{k=1}^{n}\left[1+2\left(\frac{k}{n}\right)^2\right]\left(\frac{2}{n}\right) = \lim_{n\to\infty}\left(\frac{2}{n}\right)\left[\sum_{k=1}^{n}1 + \frac{2}{n^2}\sum_{k=1}^{n}k^2\right] = \lim_{n\to\infty}\left[\left(\frac{2}{n}\right)n + \frac{2}{n}\cdot\frac{2}{n^2}\cdot\frac{n(n+1)(2n+1)}{6}\right]$$
$$= \lim_{n\to\infty}\left[2+\frac{2}{3}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right] = 2 + \frac{4}{3} = \frac{10}{3}$$

**44.**
$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{2k-1}{2n}\right)\left(\frac{1}{n}\right) = \lim_{n\to\infty}\frac{1}{n}\left[\sum_{k=1}^{n}\frac{2n+2k-1}{2n}\right] = \lim_{n\to\infty}\frac{1}{2n^2}\left[(2n-1)\sum_{k=1}^{n}1 + 2\sum_{k=1}^{n}k\right]$$
$$= \lim_{n\to\infty}\left[\frac{2n-1}{2n^2}\cdot n + \frac{1}{n^2}\cdot\frac{n(n+1)}{2}\right] = \lim_{n\to\infty}\left[1-\frac{1}{2n}+\frac{1}{2}\left(1+\frac{1}{n}\right)\right] = \frac{3}{2}$$