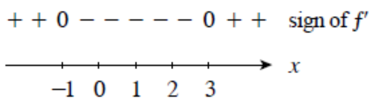
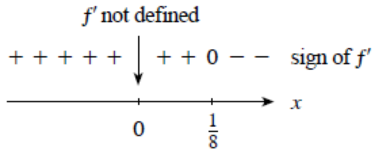
14. $f(x) = x^3 - 3x^2 - 9x + 6 \Rightarrow f'(x) = 3x^2 - 6x - 9 = 3(x-3)(x+1)$ is continuous everywhere and has zeros at -1 and 3, the critical numbers of f. The sign diagram of f' is shown.



a. f is increasing on $(-\infty, -1)$ and $(3, \infty)$ and decreasing on $(-1, 3)$.

b. f has a relative maximum of $f(-1) = 11$ and a relative minimum of $f(3) = -21$.

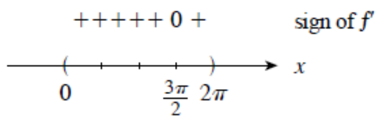
18. $f(x) = x^{1/3} - x^{2/3} \Rightarrow f'(x) = \frac{1}{3}x^{-2/3} - \frac{2}{3}x^{-1/3} = \frac{1}{3}x^{-2/3}\left(1 - 2x^{1/3}\right)$ is discontinuous at 0 and has a zero at $x = \frac{1}{8}$. The critical numbers of f are thus 0 and $\frac{1}{8}$. The sign diagram of f' is shown.



a. f is increasing on $\left(-\infty, \frac{1}{8}\right)$ and decreasing on $\left(\frac{1}{8}, \infty\right)$.

b. f has a relative maximum of $f\left(\frac{1}{8}\right) = \frac{1}{4}$.

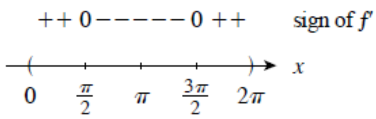
32. $f(x) = x - \cos x,\ 0 < x < 2\pi \Rightarrow f'(x) = 1 + \sin x$ is continuous on $(0, 2\pi)$ and has zeros where $1 + \sin x = 0 \Leftrightarrow \sin x = -1 \Rightarrow x = \frac{3\pi}{2}$, a critical number of f. The sign diagram of f' is shown.



a. f is increasing on $(0, 2\pi)$.

b. f has no relative extremum.

35. $f(x) = x\sin x + \cos x,\ 0 < x < 2\pi \Rightarrow f'(x) = \sin x + x\cos x - \sin x = x\cos x$ is continuous everywhere and has zeros where $x\cos x = 0 \Rightarrow x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ in $(0, 2\pi)$. The sign diagram of f' is shown.



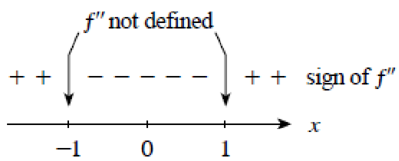
a. f is increasing on $\left(0, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$ and decreasing on $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$.

b. f has a relative maximum of $f\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$ and a relative minimum of $f\left(\frac{3\pi}{2}\right) = -\frac{3\pi}{2}$.

24. $f(x) = \dfrac{x^2 - 9}{1 - x^2} \Rightarrow f'(x) = \dfrac{\left(1 - x^2\right)(2x) - \left(x^2 - 9\right)(-2x)}{\left(1 - x^2\right)^2} = \dfrac{-16x}{\left(1 - x^2\right)^2} \Rightarrow$

$$f''(x) = \frac{\left(1 - x^2\right)^2(-16) + 16x(2)\left(1 - x^2\right)(-2x)}{\left(1 - x^2\right)^4} = -\frac{16\left(3x^2 + 1\right)}{\left(1 - x^2\right)^3}.$$

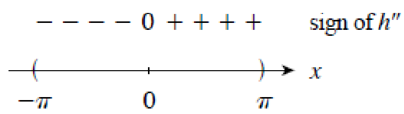
The sign diagram of f'' is shown at right. We see that f is concave upward on $(-\infty, -1)$ and $(1, \infty)$ and concave downward on $(-1, 1)$. It has no inflection point.



31. $h(x) = \dfrac{\sin x}{1 + \cos x},\ -\pi < x < \pi \Rightarrow h'(x) = \dfrac{(1 + \cos x)\cos x - \sin x(-\sin x)}{(1 + \cos x)^2} = \dfrac{1}{1 + \cos x} \Rightarrow$

$$h''(x) = \frac{d}{dx}(1 + \cos x)^{-1} = \frac{\sin x}{(1 + \cos x)^2}.$$

The sign diagram of h'' is shown at right. We see that h is concave downward on $(-\pi, 0)$ and concave upward on $(0, \pi)$. It has an inflection point at $(0, 0)$.



38. $h(x) = 2x^3 + 3x^2 - 12x - 2 \Rightarrow h'(x) = 6x^2 + 6x - 12 = 6(x+2)(x-1) = 0 \Rightarrow x = -2$ or 1, the critical numbers of h. $h''(x) = 12x + 6 = 6(2x + 1)$. We use the SDT: $h''(-2) = -18 < 0$, so h has a relative maximum of $h(-2) = 18$; and $h''(1) = 18 > 0$, so h has a relative minimum of $h(1) = -9$.

47. $f(x) = 2\sin x + \sin 2x,\ 0 < x < \pi \Rightarrow$
$f'(x) = 2\cos x + 2\cos 2x = 2\cos x + 2\left(2\cos^2 x - 1\right) = 2\left(2\cos^2 x + \cos x - 1\right) = 2(2\cos x - 1)(\cos x + 1) = 0 \Rightarrow$
$x = \frac{\pi}{3}$, the only critical number in $(0, \pi)$. $f''(x) = -2\sin x - 4\sin 2x$ and $f''\left(\frac{\pi}{3}\right) = -3\sqrt{3} < 0$, so by the SDT, f has a relative maximum of $f\left(\frac{\pi}{3}\right) = \frac{3\sqrt{3}}{2}$.