4.4

15. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{2c_k}{c_k^2+1} \Delta x = \int_1^2 \frac{2x}{x^2+1}\, dx$

16. $\lim_{n\to\infty} \sum_{k=1}^{n} c_k \cos c_k \, \Delta x = \int_0^{\pi/2} x \cos x \, dx$

47. Let $g(x) = -f(x)$. Then g is continuous and $g(x) \geq 0$ on $[a, b]$. By Property 4 of the definite integral, $0 \leq \int_a^b g(x)\, dx = \int_a^b [-f(x)]\, dx = -\int_a^b f(x)\, dx \Rightarrow \int_a^b f(x)\, dx \leq 0$.

48. $-|f(x)| \leq f(x) \leq |f(x)|$ for all x in $[a, b]$, so by Property 5 of the definite integral, $-\int_a^b |f(x)|\, dx = \int_a^b [-|f(x)|]\, dx \leq \int_a^b f(x)\, dx \leq \int_a^b |f(x)|\, dx \Rightarrow \left|\int_a^b f(x)\, dx\right| \leq \int_a^b |f(x)|\, dx$.

4.5

7. $F'(x) = \frac{d}{dx} \int_x^{\pi} \sin 2t \, dt = -\frac{d}{dx} \int_{\pi}^{x} \sin 2t \, dt = -\sin 2x$

11. $F'(x) = \frac{d}{dx} \int_1^{\cos x} \frac{t^2}{t+1}\, dt = \frac{\cos^2 x}{\cos + 1} \cdot \frac{d}{dx}(\cos x) = -\frac{\sin x \cos^2 x}{\cos x + 1}$

35. Let $u = t^2 - 1$, so $du = 2t\, dt$, $t = 1 \Rightarrow u = 0$, and $t = 2 \Rightarrow u = 3$. Then $\int_1^2 8t\left(t^2-1\right)^7 dt = 4\int_0^3 u^7\, du = \frac{1}{2}u^8\Big|_0^3 = \frac{1}{2}\left(3^8\right) = \frac{6561}{2}$.

45. Let $u = \frac{1}{x}$, so $du = -\frac{1}{x^2}\, dx \Rightarrow \frac{dx}{x^2} = -du$, $x = \frac{1}{\pi} \Rightarrow u = \pi$, and $x = \frac{2}{\pi} \Rightarrow u = \frac{\pi}{2}$. Then $\int_{1/\pi}^{2/\pi} \frac{\sin(1/x)}{x^2}\, dx = -\int_{\pi}^{\pi/2} \sin u\, du = \cos u|_{\pi}^{\pi/2} = \cos\frac{\pi}{2} - \cos\pi = 0 - (-1) = 1$.