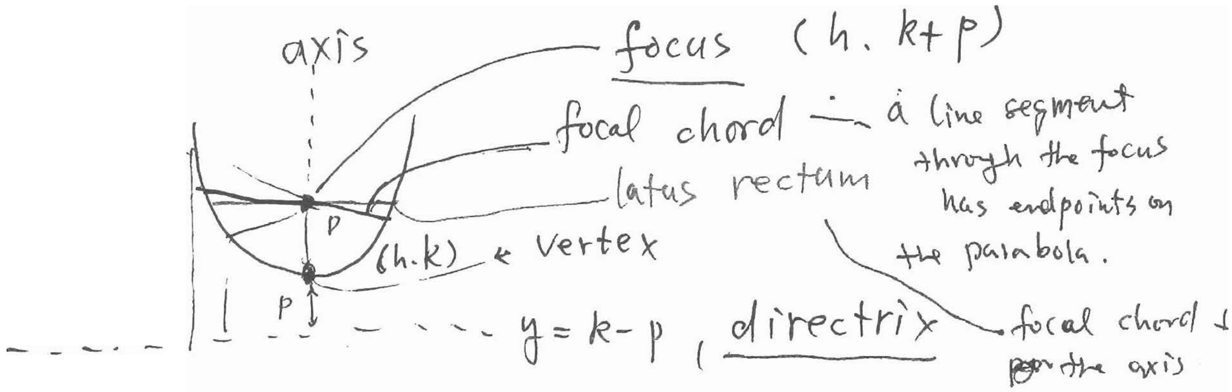
10.1

$(x-h)^2 = 4p(y-k)$ $\boxed{\text{parabola}}$



axis : $x = h$

① directrix $y = k-p$

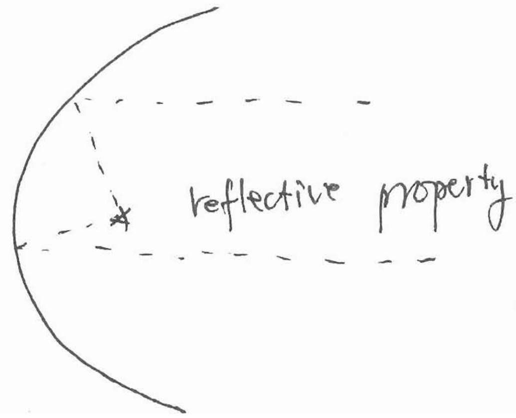
focus : $(h, k+p)$

vertex : (h, k)

~~axis~~

② focal chord

latus rectum



$$x^2 = 4py \Rightarrow y' = \frac{x}{2p}$$

~~Ellipse~~

$$s = \int_{-2p}^{2p} \left(1 + \left(\frac{x}{2p}\right)^2\right)^{\frac{1}{2}} dx = \frac{1}{p}\int_0^{2p} \sqrt{4p^2 + x^2}\, dx$$

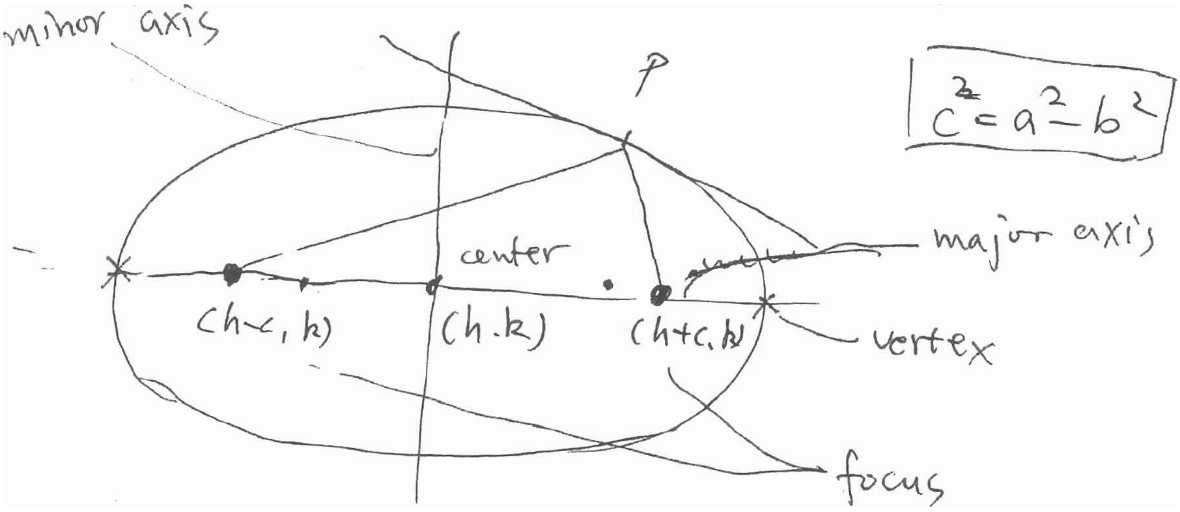
$$= \frac{1}{2p}\left[x\sqrt{4p^2+x^2} + 4p^2 \ln\left|x + \sqrt{4p^2+x^2}\right|\right]_0^{2p}$$

$$= 2p\left[\sqrt{2} + \ln(1+\sqrt{2})\right]$$

— reflective property :

Ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



foci = $(h-c, k)$, $(h+c, k)$

eccentricity = $\frac{c}{a} < 1$

area $A = 4\int_0^{\frac{b}{a}} \frac{b}{a}\sqrt{a^2-x^2}\, dx = \frac{4b}{a}\int_0^{\frac{\pi}{2}} a^2\cos^2\theta\, d\theta$

$= \pi ab$

circumferency : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

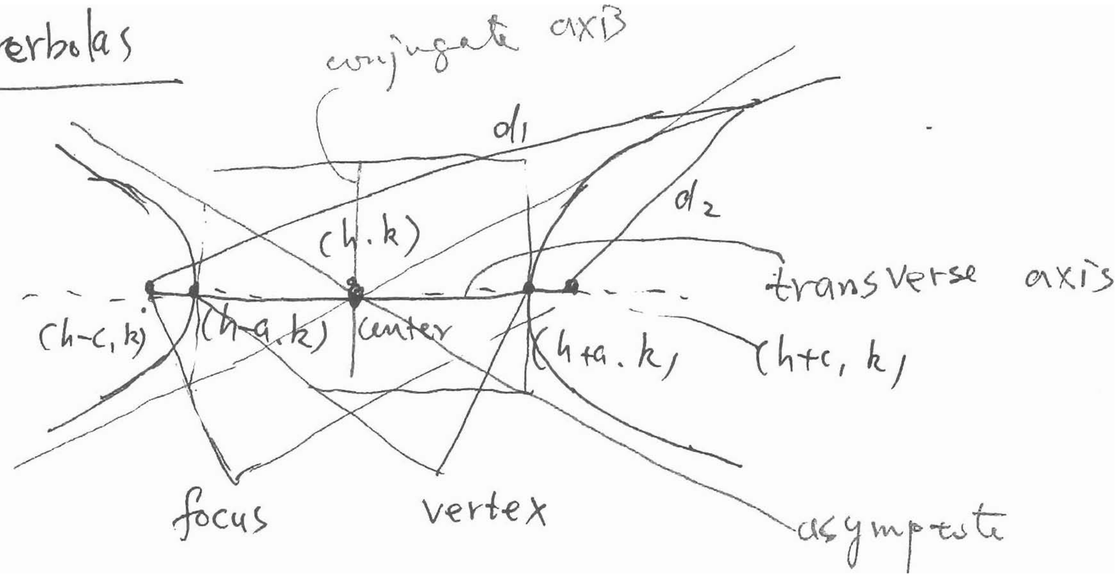
$$4a\int_0^{\pi/2} \sqrt{1-e^2\sin^2\theta}\, d\theta$$

ellipse = the set of all point (x,y) the sum of whose distance from two distinct fixed points, called foci, is constant.

reflective property = The tangent line to the ellipse at point p makes equal angles with the lines through p and the foci

10-1-2

Hyperbolas



(i) $|d_1 - d_2| = \text{const} = 2a$ $\qquad c^2 = a^2 + b^2$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

(ii) focus $(h-c.k)$, $(h+c.k)$

vertex $(h-a.k)$, $(h+a.k)$

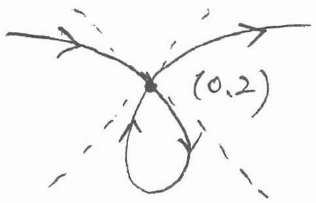
(iii) asymptotes : $y = k + \frac{b}{a}(x-h)$

$y = k - \frac{b}{a}(x+h)$

(iv) eccentricity $= \frac{c}{a} > 1$

hyperbola = the set of all points (x, y) for which the absolute value of the difference between the distances from two distinct points called foci is constant

Ex



prolate cycloid

$$\begin{cases} x = 2t - \pi \sin t \\ y = 2 - \pi \cos t \end{cases}$$

$(0,2) \Leftrightarrow t = \frac{\pi}{2}, -\frac{\pi}{2}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\pi \sin t}{2 - \pi \cos t}$$

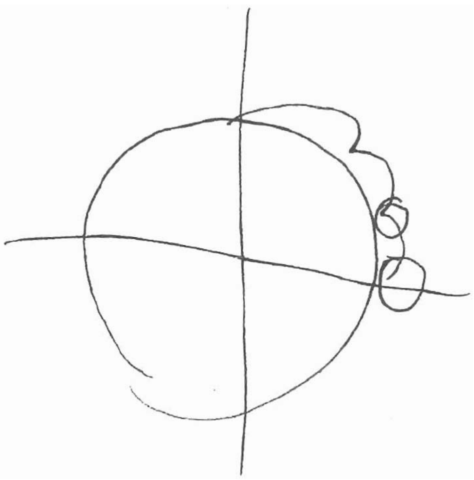
$$\left.\frac{dy}{dx}\right|_{t=-\pi/2} = -\pi/2 \Rightarrow y - 2 = -\left(\frac{\pi}{2}\right)x$$

$$\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{2}} = \pi/2 \Rightarrow y - 2 = +\left(\frac{\pi}{2}\right)x$$

Arc length

$$R = \int_{t=a}^{t=b} \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt$$

$\underline{\underline{R = 4r}}$



$x = 5\cos t - \cos 5t$

$y = 5\sin t - \sin 5t$

$$s = 4\int_0^{\pi/2} \sqrt{(-5\sin t + 5\sin 5t)^2 + (\quad)}\, dt$$

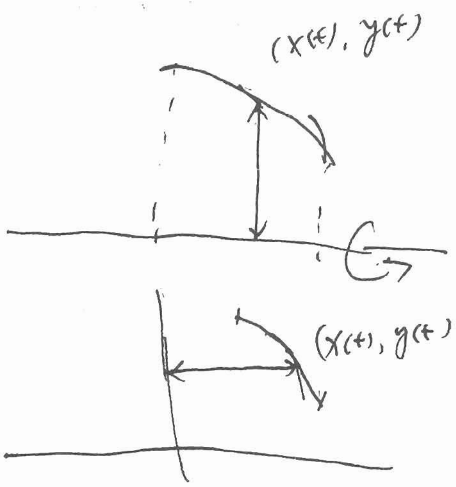
$$= 20\int_0^{\pi/2} \sqrt{2 - 2\cos 4t}\, dt$$

$$= 40$$

Rotation

Area :

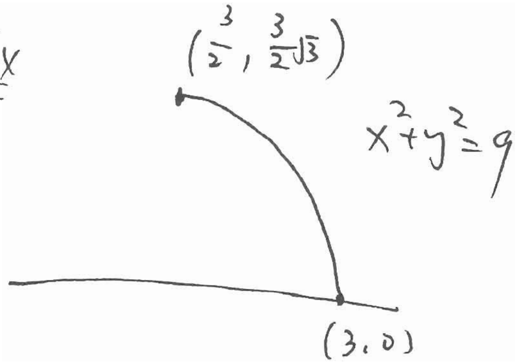
$$\begin{cases} x = f(t) \\ y = g(t) \end{cases}$$



$$\int_{t=a}^{b} 2\pi\, g(t) \cdot ds$$

$$\int_{t=c}^{t=d} 2\pi\, f(t) \cdot ds$$

Ex

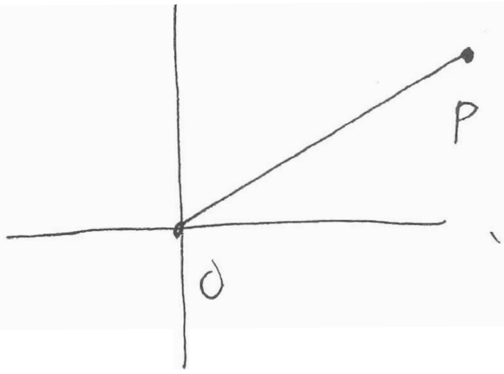


$$S = 2\pi \int_0^{\pi/3} (3\sin t)\sqrt{(-3\sin t)^2 + (3\cos t)^2}\, dt$$

$$= 6\pi \int_0^{\pi/3} 3\sin t\, dt$$

$$= 9\pi$$

10.4



$(x, y) = (r, \theta)$

O: pole (origin)

x-axis = polar axis

r = directed distance from O to P

θ = directed angle, counterclockwise from polar axis to segment $\overline{OP}$.

(r, θ) polar coordinates.

Rem ① (x, y) unique in Cartesian Coord.

~~$(r,\theta) = (r, \theta+2\pi)$, $(-r, \theta+$~~

But $(r, \theta) = (r, \theta + 2n\pi)$

$= (-r, \theta + \pi)$

$= (-r, \theta + (2n+1)\pi)$ $\quad n = \pm 0, \pm 1, \pm 2 \ldots$

the same point

② pole $= (0, \theta)$ $\quad \theta$ any angle.

Ex $r = 2(1-\cos\theta)$

$= -2(2\cos\theta+1)(\cos\theta-1)$

$\frac{dy}{d\theta} = 0 \Rightarrow \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

$\frac{dx}{d\theta} = 0 \Rightarrow \theta = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$

$= 2\sin\theta(2\cos\theta-1)$

Thm ① $\begin{cases} r^2 = x^2+y^2 \\ \tan\theta = \dfrac{y}{x} \end{cases}$

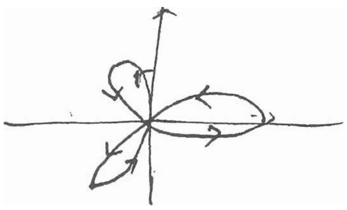
② $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

Ex ① $r = 2$

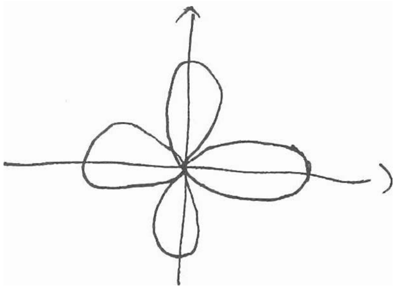
② $\theta = \dfrac{\pi}{3}$

③ $r = \sec\theta$

Ex $r = 2\cos 3\theta$



$r = 3\cos 2\theta$



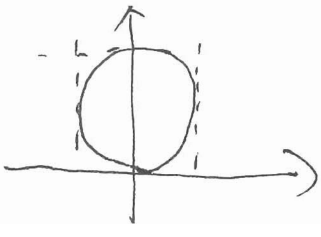
slope : $r = f(\theta)$ $\quad x = r\cos\theta = f(\theta)\cos\theta$

$y = r\sin\theta = f(\theta)\sin\theta$

$$\frac{dy}{dx} = \frac{f'\sin\theta + f\cos\theta}{f'\cos\theta - f\sin\theta}$$

Ex $r = \sin\theta$

$0 \le \theta \le \pi$

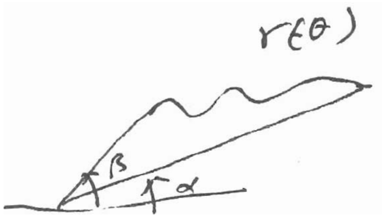


$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$, $x' = 0$

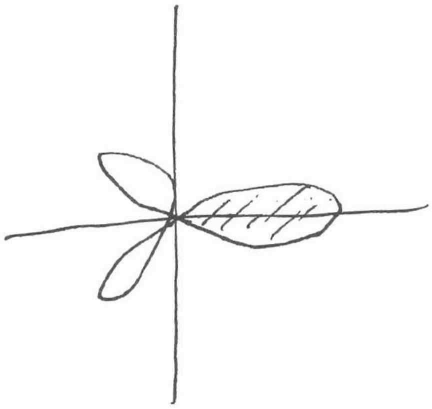
$\theta = 0, \frac{\pi}{2}$, $y' = 0$

10-5

$$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta$$



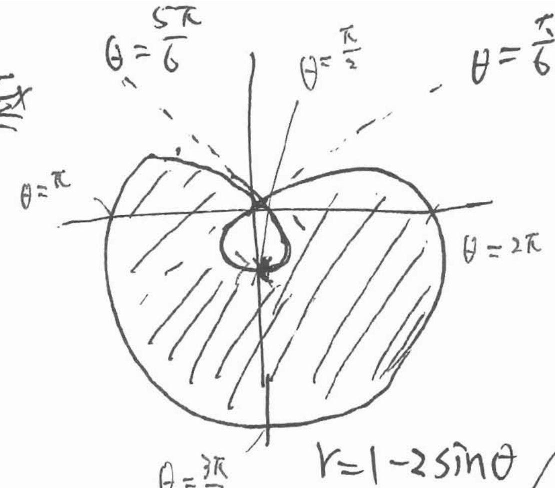
Ex



$r = 3\cos 3\theta$

$$A = \int_{\theta=-\frac{\pi}{6}}^{\theta=\frac{\pi}{6}} (3\cos 3\theta)^2 \, d\theta = \frac{3\pi}{4}$$

Ex



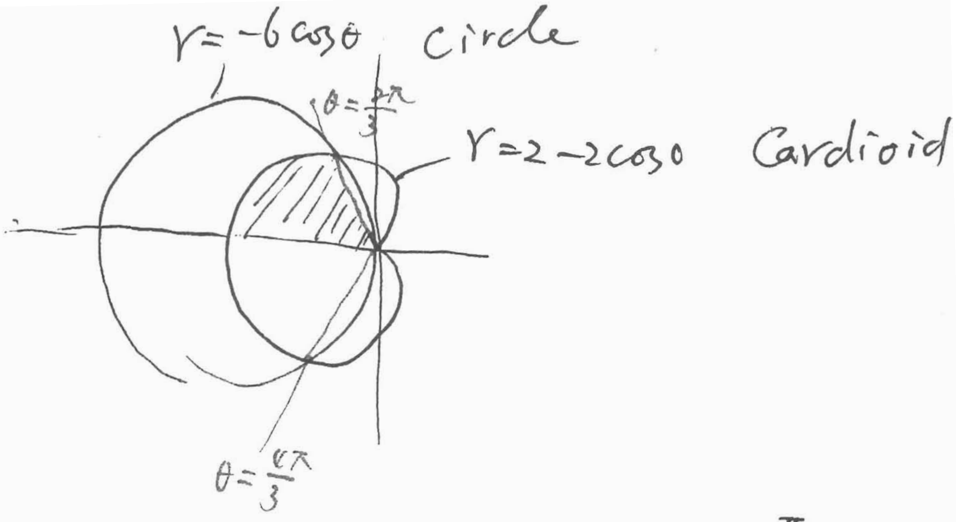
$r = 1 - 2\sin\theta$

$\theta = \frac{\pi}{6}$	$r = 0$
$\theta = \frac{\pi}{2}$	$r = -1$
$\theta = \frac{5\pi}{6}$	$r = 0$

$$A_1 = \text{linner loop} = \frac{1}{2}\int_{\theta=\frac{\pi}{6}}^{\theta=\frac{5\pi}{6}} (1-2\sin\theta)^2 \, d\theta = \pi - \frac{3\sqrt{3}}{2}$$

$$A_2 = \text{outer loop} = \frac{1}{2}\int_{\theta=\frac{5\pi}{6}}^{\theta=\frac{13\pi}{6}} (\qquad)^2 \, d\theta = 2\pi + \frac{3\sqrt{3}}{2}$$

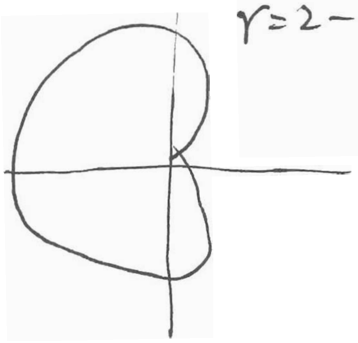
$\theta = \frac{5\pi}{6}$



$$\frac{A}{2} = \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} (-6\cos\theta)^2\, d\theta + \frac{1}{2}\int_{\theta=\frac{2\pi}{3}}^{\pi}$$

circle (0, $\frac{\pi}{2}$) has coordinate at pole..

Ex



$r = 2-2\cos\theta$

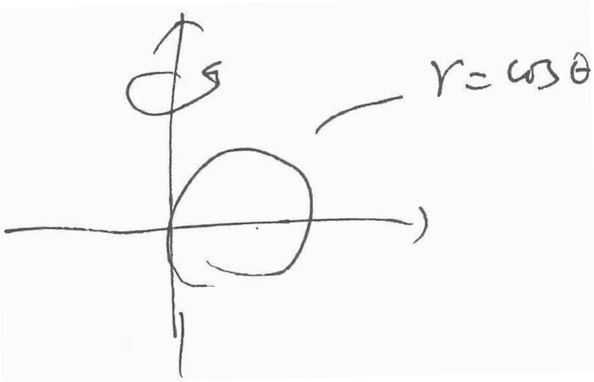
$$s = \int_{\theta=0}^{\theta=2\pi} \sqrt{(f)+(f'\theta)^2}\, d\theta = 16$$

$$x(\theta) = (2-2\cos\theta)\cos\theta \Rightarrow x'(\theta) = r'\cos\theta + r(-\sin\theta)$$

$$y(\theta) = (2-2\cos\theta)\sin\theta \Rightarrow y'(\theta) = r'\sin\theta + r\cos\theta$$

$$\Rightarrow (ds)^2 = (r^2 + r'^2)\, d\theta$$

Area



$S = \pi^2$