Week 5: 9.1: 19, 29, 37, 68

9.2: 11, 16, 29, 34

19. $\lim_{n\to\infty} \frac{2n^2 - 3n + 4}{3n^2 + 1} = \lim_{n\to\infty} \frac{2 - \frac{3}{n} + \frac{4}{n^2}}{3 + \frac{1}{n^2}} = \frac{2}{3}$

29. $\lim_{n\to\infty} \frac{\sin\sqrt{n}}{\sqrt{n}} = 0$ by the Squeeze Theorem: $-\frac{1}{\sqrt{n}} < \frac{\sin\sqrt{n}}{\sqrt{n}} < \frac{1}{\sqrt{n}}$ and $\lim_{n\to\infty}\left(\pm\frac{1}{\sqrt{n}}\right) = 0$, so the sequence converges to 0.

37. $\lim_{x\to\infty}\left[\left(1 + \frac{2}{x}\right)^{1/x}\right] = \lim_{u\to\infty}\left[\left(1 + \frac{1}{u}\right)^{1/(2u)}\right]$ (where $u = \frac{1}{2}x$) $= 1^0 = 1$

68. $a_1 = \sqrt{2} = 2^{1/2}$, $a_2 = \sqrt{2a_1} = \sqrt{2\sqrt{2}} = \sqrt{2^{3/2}} = 2^{3/4}$, $a_3 = \sqrt{2a_2} = \sqrt{2 \cdot 2^{3/4}} = 2^{7/8}, \ldots, a_n = 2^{(2^n-1)/(2^n)}$. Thus, $\lim_{n\to\infty} a_n = \lim_{n\to\infty} 2^{1-(1/2^n)} = 2^1 = 2$.

11. $\sum_{n=0}^{\infty} 2\left(-\frac{1}{\sqrt{2}}\right)^n = \frac{2}{1 - \left(-\frac{1}{\sqrt{2}}\right)} = \frac{2}{1 + \frac{1}{\sqrt{2}}} = \frac{2\sqrt{2}}{\sqrt{2}+1} \cdot \frac{\sqrt{2}-1}{\sqrt{2}-1} = 2\sqrt{2}\left(\sqrt{2}-1\right) = 2\left(2 - \sqrt{2}\right)$

16. $1 - \frac{3}{2} + \frac{9}{4} - \frac{27}{8} + \cdots = \sum_{n=1}^{\infty}\left(-\frac{3}{2}\right)^{n-1}$ is a divergent geometric series since $|r| = \left|-\frac{3}{2}\right| = \frac{3}{2} > 1$.

29. $\sum_{n=1}^{\infty} \frac{1}{n(n+2)} = \sum_{n=1}^{\infty}\left[\frac{\frac{1}{2}}{n} - \frac{\frac{1}{2}}{n+2}\right]$ is a telescoping series.

$S_n = \frac{1}{2}\left[\left(1 - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \cdots + \left(\frac{1}{n} - \frac{1}{n+2}\right)\right] = \frac{1}{2}\left(1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right)$, so

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1}{2}\left(\frac{3}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right) = \frac{3}{4}$ and so $\sum_{n=1}^{\infty} \frac{1}{n(n+2)} = \frac{3}{4}$.

34. $\sum_{n=1}^{\infty} 2^{-n}5^{n+1} = \sum_{n=1}^{\infty} 5\left(\frac{5}{2}\right)^n$ is a divergent geometric series with $|r| = \frac{5}{2} > 1$.