Week 2: 7.1: 5, 11, 15, 17

7.2: 7, 19, 22, 47

5. $\int x \ln 2x\, dx$. Let $u = \ln 2x$ and $dv = x\, dx$. Then $du = dx/x$ and $v = \int x\, dx = \frac{1}{2}x^2$, so $\int x \ln 2x\, dx = uv - \int v\, du = \frac{1}{2}x^2 \ln 2x - \int \frac{1}{2}x\, dx = \frac{1}{2}x^2 \ln 2x - \frac{1}{4}x^2 + C = \frac{1}{4}x^2 (2\ln 2x - 1) + C$.

11. $\int \tan^{-1} x\, dx$. Let $u = \tan^{-1} x$ and $dv = dx$. Then $du = \dfrac{dx}{1+x^2}$ and $v = x$, so

$\int \tan^{-1} x\, dx = uv - \int v\, du = x \tan^{-1} x - \int \dfrac{x\, dx}{1+x^2} = x \tan^{-1} x - \frac{1}{2} \ln\left(1 + x^2\right) + C$.

15. $\int x \sec^2 x\, dx$. Let $u = x$ and $dv = \sec^2 x\, dx$. Then $du = dx$ and $v = \int \sec^2 x\, dx = \tan x$, so $\int x \sec^2 x\, dx = uv - \int v\, du = x \tan x - \int \tan x\, dx = x \tan x - \int \dfrac{\sin x}{\cos x}\, dx = x \tan x + \ln|\cos x| + C$.

17. $I = \int e^{2x} \cos 3x\, dx$. Let $u = e^{2x}$ and $dv = \cos 3x\, dx$, so $du = 2e^{2x}\, dx$ and $v = \int \cos 3x\, dx = \frac{1}{3}\sin 3x$. Then $I = uv - \int v\, du = \frac{1}{3}e^{2x} \sin 3x - \frac{2}{3}\int e^{2x} \sin 3x\, dx$. To evaluate the integral on the right, let $s = e^{2x}$ and $dt = \sin 3x\, dx$, so $ds = 2e^{2x}\, dx$ and $t = \int \sin 3x\, dx = -\frac{1}{3}\cos 3x$. Then $I = \frac{1}{3}e^{2x} \sin 3x - \frac{2}{3}\left(-\frac{1}{3}e^{2x} \cos 3x + \frac{2}{3} I\right) = \frac{1}{3}e^{2x} \sin 3x + \frac{2}{9}e^{2x} \cos 3x - \frac{4}{9} I$. Thus, $\left(1 + \frac{4}{9}\right) I = \frac{1}{3}e^{2x} \sin 3x + \frac{2}{9}e^{2x} \cos 3x \Rightarrow I = \int e^{2x} \cos 3x\, dx = \frac{1}{13}e^{2x} (3\sin 3x + 2\cos 3x) + C$.

7. $\displaystyle\int_0^{\pi} \cos^2 \frac{x}{2}\, dx = \int_0^{\pi} \frac{1+\cos x}{2}\, dx = \left.\frac{x + \sin x}{2}\right|_0^{\pi} = \frac{\pi}{2}$

19. $\int_0^{\pi/4} \tan^2 x\, dx = \int_0^{\pi/4} \left(\sec^2 x - 1\right) dx = (\tan x - x)|_0^{\pi/4} = 1 - \frac{\pi}{4} = \frac{4-\pi}{4}$

22. $\int \tan^5 x \sec^3 x\, dx = \int \tan^4 x \sec^2 x \sec x \tan x\, dx = \int \left(\sec^2 x - 1\right)^2 \sec^2 x \sec x \tan x\, dx$

$= \int \left(\sec^6 x - 2\sec^4 x + \sec^2 x\right) \sec x \tan x\, dx = \frac{1}{7}\sec^7 x - \frac{2}{5}\sec^5 x + \frac{1}{3}\sec^3 x + C$

47. $\displaystyle\int \frac{1 - \tan^2 x}{\sec^2 x}\, dx = \int \frac{1 - \left(\sec^2 x - 1\right)}{\sec^2 x}\, dx = \int \frac{2 - \sec^2 x}{\sec^2 x}\, dx = \int \left(2\cos^2 x - 1\right) dx = \int \cos 2x\, dx = \frac{1}{2}\sin 2x + C$