Week 11: 10.3: 3, 7, 17, 19

11.3: 39, 45

11.4: 15, 29

3. $x = \sqrt{t},\ y = \frac{1}{t} \Rightarrow \frac{dx}{dt} = \frac{1}{2\sqrt{t}}$ and $\frac{dy}{dt} = -\frac{1}{t^2}$. The slope of the tangent line at $t = 1$ is
$\left.\frac{dy}{dx}\right|_{t=1} = \left.\frac{dy/dt}{dx/dt}\right|_{t=1} = \left.\frac{-1/t^2}{1/\left(2\sqrt{t}\right)}\right|_{t=1} = -2.$

7. $x = 2t - 1,\ y = t^3 - t^2 \Rightarrow \frac{dx}{dt} = 2$ and $\frac{dy}{dt} = 3t^2 - 2t \Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2 - 2t}{2}$. The point of tangency is $(x(1), y(1)) = (1, 0)$ and the slope of the tangent line is $\left.\frac{dy}{dx}\right|_{t=1} = \left.\frac{3t^2 - 2t}{2}\right|_{t=1} = \frac{1}{2}$, so an equation is $y - 0 = \frac{1}{2}(x - 1)$ or $y = \frac{1}{2}x - \frac{1}{2}$.

17. $x = 3t^2 + 1,\ y = 2t^3 \Rightarrow \frac{dx}{dt} = 6t$ and $\frac{dy}{dt} = 6t^2$, so $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6t^2}{6t} = t$ and $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{1}{6t}$.

19. $x = \sqrt{t},\ y = \frac{1}{t} \Rightarrow \frac{dx}{dt} = \frac{1}{2\sqrt{t}}$ and $\frac{dy}{dt} = -\frac{1}{t^2}$, so $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-1/t^2}{1/\left(2t^{1/2}\right)} = -2t^{-3/2} = -\frac{2}{t^{3/2}}$ and
$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\left(-2t^{-3/2}\right)}{1/\left(2t^{1/2}\right)} = 3t^{-5/2} \cdot 2t^{1/2} = \frac{6}{t^2}.$

39. $\mathbf{a} = \langle 2, 3\rangle$ and $\mathbf{b} = \langle 1, 4\rangle$.

a. $\text{proj}_{\mathbf{a}}\,\mathbf{b} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a} = \left(\frac{\langle 2,3\rangle \cdot \langle 1,4\rangle}{4+9}\right)\langle 2,3\rangle = \frac{2+12}{13}\langle 2,3\rangle = \left\langle \frac{28}{13}, \frac{42}{13}\right\rangle$

b. $\text{proj}_{\mathbf{b}}\,\mathbf{a} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{b}|^2}\right)\mathbf{b} = \left(\frac{\langle 2,3\rangle \cdot \langle 1,4\rangle}{1+16}\right)\langle 1,4\rangle = \frac{2+12}{17}\langle 1,4\rangle = \left\langle \frac{14}{17}, \frac{56}{17}\right\rangle$

45. $\mathbf{a} = \langle 1, 3\rangle$ and $\mathbf{b} = \langle 2, 4\rangle$. Because $\text{proj}_{\mathbf{a}}\,\mathbf{b} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a} = \left(\frac{\langle 1,3\rangle \cdot \langle 2,4\rangle}{1+9}\right)\langle 1,3\rangle = \frac{2+12}{10}\langle 1,3\rangle = \left\langle \frac{7}{5}, \frac{21}{5}\right\rangle$ is parallel to $\mathbf{a}$, we can write $\mathbf{b} = \text{proj}_{\mathbf{a}}\,\mathbf{b} + \left(\mathbf{b} - \text{proj}_{\mathbf{a}}\,\mathbf{b}\right) = \left\langle \frac{7}{5}, \frac{21}{5}\right\rangle + \left\langle \frac{3}{5}, -\frac{1}{5}\right\rangle$.

15. $\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 1 & -2 \\ 1 & 1 & 1 \end{vmatrix} = 3\mathbf{i} + \mathbf{j} - 4\mathbf{k}$, so the required unit vectors are $\pm\left(\frac{3\mathbf{i} + \mathbf{j} - 4\mathbf{k}}{\sqrt{9+1+16}}\right)$; that is, $\pm\frac{\sqrt{26}}{26}(3\mathbf{i} + \mathbf{j} - 4\mathbf{k})$.

29. For $P(0,0,0)$, $Q(3,-2,1)$, $R(1,2,2)$, and $S(1,1,4)$, $\overrightarrow{PQ} = \langle 3,-2,1\rangle$, $\overrightarrow{PR} = \langle 1,2,2\rangle$, and $\overrightarrow{PS} = \langle 1,1,4\rangle$, so
$\overrightarrow{PQ}\cdot\left(\overrightarrow{PR}\times\overrightarrow{PS}\right) = \begin{vmatrix} 3 & -2 & 1 \\ 1 & 2 & 2 \\ 1 & 1 & 4 \end{vmatrix} = 21$. Thus, $V = \left|\overrightarrow{PQ}\cdot\left(\overrightarrow{PR}\times\overrightarrow{PS}\right)\right| = |21| = 21$.