• (Section 13.2)

2. Along $y = 0$, $\lim_{(x,y)\to(0,0)} \frac{2x^2 - 3xy + 4y^2}{2x^2 + 3y^2} = \lim_{x\to 0} \frac{2x^2}{2x^2} = \lim_{x\to 0} 1 = 1$. Along $x = 0$, $\lim_{(x,y)\to(0,0)} \frac{2x^2 - 3xy + 4y^2}{2x^2 + 3y^2} = \lim_{y\to 0} \frac{4y^2}{3y^2} = \lim_{y\to 0} \frac{4}{3} = \frac{4}{3}$. Because these two limits are not equal, the given limit does not exist.

10. Along the y-axis, $\lim_{(x,y,z)\to(0,0,0)} \frac{2xyz}{x^3 + y^3 + z^3} = \lim_{y\to 0} \frac{0}{y^3} = \lim_{y\to 0} 0 = 0$. Let C denote the straight line with parametric equations $x = t, y = t, z = t$. Then along C, $\lim_{(x,y,z)\to(0,0,0)} \frac{2xyz}{x^3 + y^3 + z^3} = \lim_{t\to 0} \frac{2t^3}{3t^3} = \lim_{t\to 0} \frac{2}{3} = \frac{2}{3}$. Because these two limits are not equal, the given limit does not exist.

20. $\lim_{(x,y)\to(0,1)} \frac{\sin^{-1}(x/y)}{1 + (x/y)} = \frac{\sin^{-1} 0}{1 + 0} = 0$

31. The function f is a rational function and is continuous at all points where its denominator is nonzero. Thus, f is continuous on $\{(x, y) \mid 2x + 3y \neq 1\}$.

• (Section 13.3)

12. $h_u(u, v) = \frac{\partial}{\partial u} \ln\left(u^2 + v^2\right) = \frac{\frac{\partial}{\partial u}\left(u^2 + v^2\right)}{u^2 + v^2} = \frac{2u}{u^2 + v^2}$ and by symmetry, $h_v(u, v) = \frac{2v}{u^2 + v^2}$.

30. $\frac{\partial}{\partial x}\left(x^2 y + xz + yz^2\right) = \frac{\partial}{\partial x}(8) \Rightarrow 2xy + z + x\frac{\partial z}{\partial x} + 2yz\frac{\partial z}{\partial x} = 0 \Rightarrow \frac{\partial z}{\partial x} = -\frac{2xy + z}{x + 2yz}$ and $\frac{\partial}{\partial y}\left(x^2 y + xz + yz^2\right) = \frac{\partial}{\partial y}(8) \Rightarrow x^2 + x\frac{\partial z}{\partial y} + z^2 + 2yz\frac{\partial z}{\partial y} = 0 \Rightarrow \frac{\partial z}{\partial y} = -\frac{x^2 + z^2}{x + 2yz}$.

36. $\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left(xe^{2y} + ye^{2x}\right) = e^{2y} + 2ye^{2x}$, $\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(xe^{2y} + ye^{2x}\right) = 2xe^{2y} + e^{2x}$, $\frac{\partial^2 z}{\partial x^2}(x, y) = \frac{\partial}{\partial x}\left(e^{2y} + 2ye^{2x}\right) = 4ye^{2x}$, $\frac{\partial^2 z}{\partial y^2}(x, y) = \frac{\partial}{\partial y}\left(2xe^{2y} + e^{2x}\right) = 4xe^{2y}$, $\frac{\partial^2 z}{\partial x\, \partial y} = \frac{\partial}{\partial x}\left(e^{2x} + 2xe^{2y}\right) = 2e^{2x} + 2e^{2y}$, and $\frac{\partial^2 z}{\partial y\, \partial x} = \frac{\partial}{\partial y}\left(e^{2y} + 2ye^{2x}\right) = 2e^{2x} + 2e^{2y}$.

52. $f_x(x, y, z) = \frac{\partial}{\partial x}\left[\ln(x + 2y + 3z)\right] = \frac{1}{x + 2y + 3z} \Rightarrow f_{xy}(x, y, z) = -2(x + 2y + 3z)^{-2} \Rightarrow$ $f_{xyz}(x, y, z) = \frac{12}{(x + 2y + 3z)^3}$, $f_y(x, y, z) = \frac{2}{x + 2y + 3z} \Rightarrow f_{yx}(x, y, z) = -2(x + 2y + 3z)^{-2} \Rightarrow$ $f_{yxz}(x, y, z) = \frac{12}{(x + 2y + 3z)^3}$, and $f_z(x, y, z) = \frac{3}{x + 2y + 3z} \Rightarrow f_{zy}(x, y, z) = -6(x + 2y + 3z)^{-2} \Rightarrow$ $f_{zyx}(x, y, z) = \frac{12}{(x + 2y + 3z)^3}$, so $f_{xyz} = f_{yxz} = f_{zyx}$.

67. $\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left[\left(x^2 + y^2\right)^{1/2} \tan^{-1}\frac{y}{x}\right] = x\left(x^2 + y^2\right)^{-1/2} \tan^{-1}\frac{y}{x} + \left(x^2 + y^2\right)^{1/2} \cdot \frac{-y/x^2}{1 + (y/x)^2} = \frac{x\tan^{-1}(y/x) - y}{\sqrt{x^2 + y^2}}$ and $\frac{\partial z}{\partial y} = y\left(x^2 + y^2\right)^{-1/2} \tan^{-1}\frac{y}{x} + \left(x^2 + y^2\right)^{1/2} \cdot \frac{1/x}{1 + (y/x)^2} = \frac{y\tan^{-1}(y/x) + x}{\sqrt{x^2 + y^2}}$, so $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{x^2\tan^{-1}(y/x) - xy}{\sqrt{x^2 + y^2}} + \frac{y^2\tan^{-1}(y/x) + xy}{\sqrt{x^2 + y^2}} = \sqrt{x^2 + y^2}\tan^{-1}\frac{y}{x} = z$, as was to be shown.