2.5 Derivatives of Trigonometric Functions

7. $s' = \dfrac{d}{dx}(\sin x \cos x) = \sin x \dfrac{d}{dx}(\cos x) + \cos x \dfrac{d}{dx}(\sin x) = -\sin^2 x + \cos^2 x = \cos^2 x - \sin^2 x = \cos 2x$

11. $g'(x) = \dfrac{d}{dx}\left(\dfrac{\sin x}{x}\right) = \dfrac{x\cos x - \sin x}{x^2}$

19. $h'(\theta) = \dfrac{d}{d\theta}\left(\dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}\right) = \dfrac{(\sin\theta - \cos\theta)\dfrac{d}{d\theta}(\sin\theta + \cos\theta) - (\sin\theta + \cos\theta)\dfrac{d}{d\theta}(\sin\theta - \cos\theta)}{(\sin\theta - \cos\theta)^2}$

$= \dfrac{(\sin\theta - \cos\theta)(\cos\theta - \sin\theta) - (\sin\theta + \cos\theta)(\cos\theta + \sin\theta)}{(\sin\theta - \cos\theta)^2}$

$= \dfrac{\sin\theta\cos\theta - \sin^2\theta - \cos^2\theta + \sin\theta\cos\theta - \sin\theta\cos\theta - \sin^2\theta - \cos^2\theta - \sin\theta\cos\theta}{(\sin\theta - \cos\theta)^2} = -\dfrac{2}{(\sin\theta - \cos\theta)^2}$

22. $y' = \dfrac{d}{dt}\left(\dfrac{a\sin t}{1 + b\cos t}\right) = \dfrac{(1 + b\cos t)\dfrac{d}{dt}(a\sin t) - (a\sin t)\dfrac{d}{dt}(1 + b\cos t)}{(1 + b\cos t)^2}$

$= \dfrac{(1 + b\cos t)(a\cos t) - (a\sin t)(-b\sin t)}{(1 + b\cos t)^2} = \dfrac{a(\cos t + b)}{(1 + b\cos t)^2}$

28. $\dfrac{dw}{d\theta} = \dfrac{d}{d\theta}\left(\dfrac{\cos\theta}{\theta}\right) = \dfrac{\theta\dfrac{d}{d\theta}(\cos\theta) - \cos\theta\dfrac{d}{d\theta}(\theta)}{\theta^2} = -\dfrac{\theta\sin\theta + \cos\theta}{\theta^2} \Rightarrow$

$\dfrac{d^2w}{d\theta^2} = -\dfrac{\theta^2\dfrac{d}{d\theta}(\theta\sin\theta + \cos\theta) - (\theta\sin\theta + \cos\theta)\dfrac{d}{d\theta}\left(\theta^2\right)}{\theta^4}$

$= -\dfrac{\theta^2(\sin\theta + \theta\cos\theta - \sin\theta) - (\theta\sin\theta + \cos\theta)(2\theta)}{\theta^4} = \dfrac{2\cos\theta + 2\theta\sin\theta - \theta^2\cos\theta}{\theta^3}$

2.6 The Chain Rule

18. $f(x) = \dfrac{x}{\sqrt{x^2 - x - 2}} = x\left(x^2 - x - 2\right)^{-1/2} \Rightarrow$

$f'(x) = x\left(-\frac{1}{2}\right)\left(x^2 - x - 2\right)^{-3/2}(2x - 1) + \left(x^2 - x - 2\right)^{-1/2}$

$= \frac{1}{2}\left(x^2 - x - 2\right)^{-3/2}\left[-x(2x - 1) + 2\left(x^2 - x - 2\right)\right] = -\dfrac{x + 4}{2\left(x^2 - x - 2\right)^{3/2}}$

24. $y = \dfrac{(t + 1)^3}{\left(t^2 + 2t\right)^2} = (t + 1)^3\left(t^2 + 2t\right)^{-2} \Rightarrow$

$\dfrac{dy}{dt} = (t + 1)^3(-2)\left(t^2 + 2t\right)^{-3}(2t + 2) + \left(t^2 + 2t\right)^{-2}(3)(t + 1)^2$

$= (t + 1)^2\left(t^2 + 2t\right)^{-3}\left[-4(t + 1)^2 + 3\left(t^2 + 2t\right)\right] = (t + 1)^2\left(t^2 + 2t\right)^{-3}\left(-4t^2 - 8t - 4 + 3t^2 + 6t\right)$

$= -\dfrac{\left(t^2 + 2t + 4\right)(t + 1)^2}{t^3(t + 2)^3}$

39. $y = \sqrt{1 + 2\cos x} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2\sqrt{1 + 2\cos x}}\dfrac{d}{dx}(1 + 2\cos x) = -\dfrac{\sin x}{\sqrt{1 + 2\cos x}}$

42. $y = \dfrac{x + \sin 2x}{2 + \cos 3x} \Rightarrow$

$$y' = \frac{(2 + \cos 3x)(1 + 2\cos 2x) - (x + \sin 2x)(-3\sin 3x)}{(2 + \cos 3x)^2}$$
$$= \frac{2 + 4\cos 2x + \cos 3x + 2\cos 2x \cos 3x + 3x \sin 3x + 3\sin 2x \sin 3x}{(2 + \cos 3x)^2}$$

53. $f(x) = \sin(\sin x) \Rightarrow f'(x) = \cos(\sin x)\dfrac{d}{dx}(\sin x) = \cos x \cos(\sin x)$

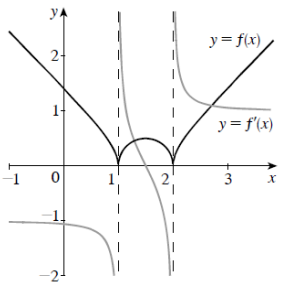
73. $F'(x) = \frac{d}{dx}\left[f\left(x^2+1\right) + g\left(x^2-1\right)\right] = \frac{d}{dx}f\left(x^2+1\right) + \frac{d}{dx}g\left(x^2-1\right)$

$$= f'\left(x^2+1\right)\tfrac{d}{dx}\left(x^2+1\right) + g'\left(x^2-1\right)\tfrac{d}{dx}\left(x^2-1\right) = 2xf'\left(x^2+1\right) + 2xg'\left(x^2-1\right)$$
$$= 2x\left[f'\left(x^2+1\right) + g'\left(x^2-1\right)\right]$$

105. a. $f(x) = \sqrt{|(x-1)(x-2)|} \Rightarrow$

$$f'(x) = \frac{1}{2\sqrt{|(x-1)(x-2)|}}\frac{d}{dx}\left|x^2 - 3x + 2\right|$$
$$= \frac{1}{2\sqrt{|(x-1)(x-2)|}}\frac{\left(x^2-3x+2\right)\frac{d}{dx}\left(x^2-3x+2\right)}{\left|x^2-3x+2\right|}$$
$$= \frac{(2x-3)(x-1)(x-2)}{2\left|x^2-3x+2\right|^{3/2}},\ x \neq 1, 2$$

b.



2.7 Implicit Differentiation

3. $xy^2 + yx^2 - 2 = 0 \Rightarrow y^2 + 2xyy' + y'x^2 + 2xy = 0 \Rightarrow \left(2xy + x^2\right)y' = -\left(y^2 + 2xy\right) \Rightarrow y' = -\dfrac{y(y+2x)}{x(2y+x)}$

17. $\tan^2\left(x^3+y^3\right) = xy \Rightarrow \left[2\tan\left(x^3+y^3\right)\sec^2\left(x^3+y^3\right)\right]\left(3x^2 + 3y^2y'\right) = y + xy' \Rightarrow$

$$\left[6y^2\tan\left(x^3+y^3\right)\sec^2\left(x^3+y^3\right) - x\right]y' = y - 6x^2\tan\left(x^3+y^3\right)\sec^2\left(x^3+y^3\right) \Rightarrow$$
$$y' = \frac{y - 6x^2\tan\left(x^3+y^3\right)\sec^2\left(x^3+y^3\right)}{6y^2\tan\left(x^3+y^3\right)\sec^2\left(x^3+y^3\right) - x}$$

21. $x^2 + 4y^2 = 4 \Rightarrow 2x + 8yy' = 0 \Rightarrow y' = -\dfrac{x}{4y}$, so $y'\big|_{\left(1,-\sqrt{3}/2\right)} = -\dfrac{1}{4\left(-\frac{\sqrt{3}}{2}\right)} = \dfrac{\sqrt{3}}{6}$. An equation of the tangent line is

$y - \left(-\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{6}(x-1)$ or $y = \frac{\sqrt{3}}{6}x - \frac{2\sqrt{3}}{3}$.

35. $y^2 - xy^2 - x^3 = 0 \Rightarrow 2yy' - y^2 - 2xyy' - 3x^2 = 0$. At $\left(\frac{1}{2}, \frac{1}{2}\right)$, $2\left(\frac{1}{2}\right)y' - \left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)y' - 3\left(\frac{1}{2}\right)^2 = 0 \Rightarrow$

$y' = 2$, so an equation of the tangent line is $y - \frac{1}{2} = 2\left(x - \frac{1}{2}\right)$ or $y = 2x - \frac{1}{2}$.

57. True. Differentiating both sides of the equation with respect to x, we have $\dfrac{d}{dx}\left[f(x)g(y)\right] = \dfrac{d}{dx}(0) \Rightarrow$

$f(x)g'(y)\dfrac{dy}{dx} + f'(x)g(y) = 0 \Rightarrow \dfrac{dy}{dx} = -\dfrac{f'(x)g(y)}{f(x)g'(y)}$, provided $f(x) \neq 0$ and $g'(y) \neq 0$.