1. (15%) Find the relative extrema of $f(x) = 15x^{2/3} - 3x^{5/3}$.

2. (15%) Sketch the graph of the function $f(x) = \frac{x^2}{x^2-1}$.

3. (15%) Find $\frac{dy}{dx}$ at the point $\left(\frac{\pi}{2}, \pi\right)$ if $x \sin y - y \cos 2x = 2x$.

4. (a) (5%) Find $\lim_{x \to -3} \frac{x^2+2x-3}{x^2+4x+3}$.

    (b) (5%) Find $\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x}$.

    (c) (5%) Let
    $$f(x) = \begin{cases} -x+3 & \text{if } x < 2 \\ \sqrt{x-2}+1 & \text{if } x \geq 2 \end{cases}$$

    Find $\lim_{x \to 2} f(x)$ if it exists.

5. (10%) Let $h(x) = \frac{\sin x}{1+\cos x}$, $-\pi < x < \pi$. Determine where the graph of $h(x)$ is concave upward, where $h(x)$ is concave downward. Also, find all inflection points of $h(x)$.

6. (10%) Prove that $f(x) = x^5 + 6x + 4$ has exactly one zero in $(-\infty, \infty)$.

7. (10%) Find the extreme values of the function $f(x) = 3x^4 - 4x^3 - 8$ on $[-1, 2]$.

8. Find $\frac{dy}{dx}$ if

    (a) (5%) $y = \tan^3(3x^2 + 1)$,

    (b) (5%) $y = \left(\frac{2x-1}{x^2+1}\right)^5$.