4. 5

11. $F'(x) = \dfrac{d}{dx}\displaystyle\int_1^{\cos x} \dfrac{t^2}{t+1}\,dt = \dfrac{\cos^2 x}{\cos +1} \cdot \dfrac{d}{dx}(\cos x) = -\dfrac{\sin x \cos^2 x}{\cos x + 1}$

12. $G'(x) = \dfrac{d}{dx}\displaystyle\int_{\sqrt{x}}^{5} \dfrac{\sin t^2}{t}\,dt = -\dfrac{d}{dx}\displaystyle\int_5^{x^{1/2}} \dfrac{\sin t^2}{t}\,dt = -\dfrac{\sin x}{\sqrt{x}} \cdot \dfrac{d}{dx}\left(x^{1/2}\right) = -\dfrac{\sin x}{2x}$

16. $\int_0^2 \left(2 - 4u + u^2\right) du = 2u - 2u^2 + \frac{1}{3}u^3 \Big|_0^2 = \left(4 - 8 + \frac{8}{3}\right) - 0 = -\frac{4}{3}$

47. $\displaystyle\int_0^3 \frac{x\,dx}{\sqrt{x+1} + \sqrt{5x+1}} = \int_0^3 \frac{x}{\sqrt{x+1} + \sqrt{5x+1}} \frac{\sqrt{x+1} - \sqrt{5x+1}}{\sqrt{x+1} - \sqrt{5x+1}}\,dx = \int_0^3 \frac{x\left(\sqrt{x+1} - \sqrt{5x+1}\right)}{(x+1) - (5x+1)}\,dx$

$= -\frac{1}{4}\int_0^3 \sqrt{x+1}\,dx + \frac{1}{4}\int_0^3 \sqrt{5x+1}\,dx$

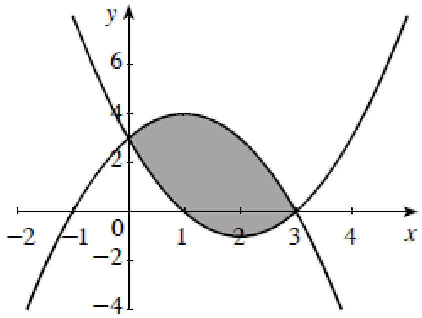
To evaluate $\int \sqrt{x+1}\,dx$, let $u = x + 1$, so $du = dx$, $x = 0 \Rightarrow u = 1$, and $x = 3 \Rightarrow u = 4$. Then $-\frac{1}{4}\int_0^3 \sqrt{x+1}\,dx = -\frac{1}{4}\int_1^4 u^{1/2}\,du = -\frac{1}{4}\left(\frac{2}{3}u^{3/2}\right)\Big|_1^4 = -\frac{1}{6}(8-1) = -\frac{7}{6}$. To evaluate the other integral, let $u = 5x + 1$, so $du = 5\,dx \Rightarrow dx = \frac{1}{5}\,du$, $x = 0 \Rightarrow u = 1$, and $x = 3 \Rightarrow u = 16$. Thus, $\frac{1}{4}\int_0^3 \sqrt{5x+1}\,dx = \frac{1}{20}\int_1^{16} u^{1/2}\,du = \frac{1}{20}\left(\frac{2}{3}u^{3/2}\right)\Big|_1^{16} = \frac{1}{30}(64-1) = \frac{21}{10}$. We conclude that $\displaystyle\int_0^3 \frac{x\,dx}{\sqrt{x+1} + \sqrt{5x+1}} = -\frac{7}{6} + \frac{21}{10} = \frac{14}{15}$.

88. $\dfrac{d}{dy}\left[\int_0^x \sqrt{3 + 2\cos t}\,dt + \int_0^y \sin t\,dt\right] = \dfrac{d}{dy}(0) = 0 \Rightarrow \sqrt{3 + 2\cos x}\dfrac{dx}{dy} + \sin y = 0 \Rightarrow \dfrac{dx}{dy} = -\dfrac{\sin y}{\sqrt{3 + 2\cos x}}$

5. 1

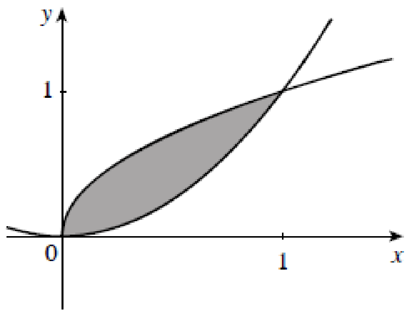
13. Solve $x^2 - 4x + 3 = -x^2 + 2x + 3 \Leftrightarrow 2x^2 - 6x = 2x(x-3) = 0$, giving $(0, 3)$ and $(3, 0)$ as the points of intersection. Thus,

$$A = \int_0^3 \left[\left(-x^2 + 2x + 3\right) - \left(x^2 - 4x + 3\right)\right] dx$$
$$= \int_0^3 \left(-2x^2 + 6x\right) dx = -\tfrac{2}{3}x^3 + 3x^2 \Big|_0^3 = 9$$



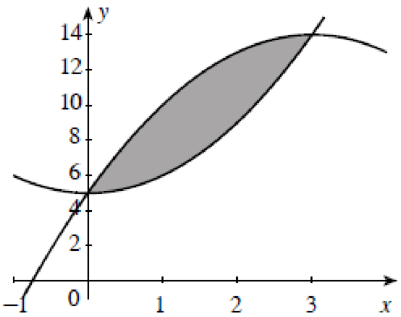
17. Solve $\sqrt{x} = x^2 \Leftrightarrow x = x^4 \Leftrightarrow x\left(x^3 - 1\right) = 0$, giving $(0, 0)$ and $(1, 1)$ as the points of intersection. Thus,

$$A = \int_0^1 \left(\sqrt{x} - x^2\right) dx = \tfrac{2}{3}x^{3/2} - \tfrac{1}{3}x^3 \Big|_0^1 = \tfrac{1}{3}.$$



23. Solve $x^2 + 5 = -x^2 + 6x + 5 \Leftrightarrow 2x^2 - 6x = 0 \Leftrightarrow 2x(x-3) = 0$, giving $(0, 5)$ and $(3, 14)$ as the points of intersection. Thus,

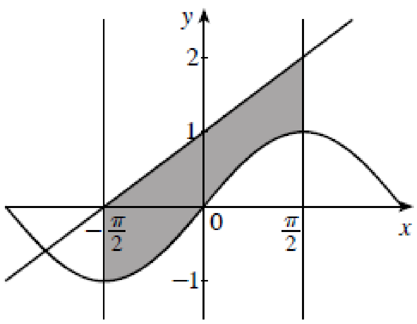
$$A = \int_0^3 \left[\left(-x^2 + 6x + 5\right) - \left(x^2 + 5\right)\right] dx = \int_0^3 \left(-2x^2 + 6x\right) dx$$
$$= -\tfrac{2}{3}x^3 + 3x^2 \Big|_0^3 = 9$$



32. $A = \int_{-\pi/2}^{\pi/2} \left[\left(\frac{2}{\pi}x + 1\right) - \sin x\right] dx$

$$= 2\int_0^{\pi/2} 1\, dx = \pi$$

because $f(x) = \frac{2}{\pi}x - \sin x$ is odd.



35. $A = 2\int_0^{\pi/4} \left(2 - \sec^2 x\right) dx = 2(2x - \tan x)\big|_0^{\pi/4} = \pi - 2$