# 6.3

**19.** $\lim\limits_{x\to(\pi/2)^-} \dfrac{2e^{\tan x}}{2x-\pi} = -\infty$ since $\dfrac{2e^{\tan x}}{2x-\pi}$ is negative for $0 < x < \frac{\pi}{2}$ and $e^{\tan x} \to \infty$, while $2x - \pi \to 0$ as $x \to \left(\frac{\pi}{2}\right)^-$.

**24.** $f'(x) = \dfrac{d}{dx}\left(x^2e^{-2x}\right) = x^2\dfrac{d}{dx}\left(e^{-2x}\right) + e^{-2x}\dfrac{d}{dx}\left(x^2\right) = -2x^2e^{-2x} + 2xe^{-2x} = 2xe^{-2x}(1-x)$

**27.** $f'(x) = \dfrac{d}{dx}\left(e^x + e^{-x}\right)^{1/2} = \frac{1}{2}\left(e^x + e^{-x}\right)^{-1/2}\dfrac{d}{dx}\left(e^x + e^{-x}\right) = \frac{1}{2}\left(e^x + e^{-x}\right)^{-1/2}\left(e^x - e^{-x}\right) = \dfrac{e^x - e^{-x}}{2\sqrt{e^x + e^{-x}}}$

**42.** $e^{xy} - x^2 + y^2 = 5 \Rightarrow e^{xy}\left(y + xy'\right) - 2x + 2yy' = 0 \Leftrightarrow y'\left(xe^{xy} + 2y\right) + ye^{xy} - 2x = 0 \Leftrightarrow y' = \dfrac{2x - ye^{xy}}{xe^{xy} + 2y}$

**49.** $y = xe^{-x} \Rightarrow y' = e^{-x} - xe^{-x} = (1-x)e^{-x} \Rightarrow y'\big|_1 = 0$. Thus, the slope of the required tangent line is $m = 0$, and an equation of the line is $y - e^{-1} = 0(x-1)$ or $y = 1/e$.

## 6. 4

**23.** $y = x\left(5^{3x}\right) \Rightarrow y' = 1 \cdot 5^{3x} + x \cdot \ln 5\,(3)\left(5^{3x}\right) = [1 + (3\ln 5)\,x]\,5^{3x}$

**31.** $y = 2^{\cot x} \Rightarrow y' = (\ln 2)\,2^{\cot x}\left(-\csc^2 x\right) = -\ln 2\left(\csc^2 x\right)2^{\cot x}$

**33.** $f(x) = \log_2\left(x^2 + x + 1\right) = \dfrac{\ln\left(x^2 + x + 1\right)}{\ln 2} \Rightarrow f'(x) = \dfrac{1}{\ln 2} \cdot \dfrac{2x + 1}{x^2 + 2x + 1} = \dfrac{2x + 1}{\left(x^2 + x + 1\right)\ln 2}$

**41.** $y = \left(\sqrt{\cos x}\right)^x \Rightarrow \ln y = \ln\left(\sqrt{\cos x}\right)^x = x \ln\sqrt{\cos x} \Rightarrow \dfrac{y'}{y} = \ln\sqrt{\cos x} + x \cdot \dfrac{\frac{1}{2}(\cos x)^{-1/2}(-\sin x)}{\sqrt{\cos x}} \Rightarrow$
$y' = \dfrac{\cos x \ln(\cos x) - x \sin x}{2\cos x}\left(\sqrt{\cos x}\right)^x$

**48.** Let $u = \sqrt{x}$, so $du = \dfrac{1}{2\sqrt{x}}\,dx$, $x = 1 \Rightarrow u = 1$, and $x = 4 \Rightarrow u = 2$. Thus, $\displaystyle\int_1^4 \frac{3^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int_1^2 3^u\,du = 2 \cdot \left.\frac{3^u}{\ln 3}\right|_1^2 = \frac{12}{\ln 3}$.